Analytical approach for CRLH based antennas design

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Abstract

This paper presents an analytical model for CRLH (Composite Right-Left Handed) antennas preliminary design. The objective of this work is to develop a tool to estimate quickly the scattering and radiating characteristics of these CRLH based structures. The analytical model provides thus a set of parameters which roughly fulfill desired requirements. Afterwards, the antenna designer can refine the CRLH based antenna performances with full 3D electromagnetic simulation software.

1. Introduction

Composite Right-Left Handed (CRLH) radiating structures are very attractive antennas because of their scanning capability and wideband performances. These structures are generally periodic and present multiple resonances. In addition, when interdigital capacitors are used in the unit cell implementation, the small gap between digits requires fine meshing in the electromagnetic simulation. This could lead to very long simulation duration. In order to obtain a first design very quickly, an analytical model describing the electromagnetic behavior of these structures would be useful. The model can be used to realize antennas composed by CRLH unit cells by exploiting their radiation properties. Only based on the electromagnetic properties of the CRLH radiating structure, this model can estimate S-parameters and radiation performances of the whole antenna. To validate the analytical model, a meander antenna is designed with two branches composed by CRLH radiating structures. First of all S-parameters and radiations patterns of a linear CRLH radiating structure are presented. Afterwards the analytical model is detailed. And finally this model is used to predict S-parameters and radiations patterns of the CRLH based meander antenna.

2. Linear CRLH radiating structure

A CRLH structure is an artificial periodic transmission line structure. The CRLH unit cell used in this paper is a microstrip structure composed by an interdigital capacitor and a via-shorted stub [1] (figure 1).

The CRLH unit cell is printed on a Rogers Duroid RT5880 substrate (ε_r=2.2, tanδ=0.0009). Geometrical parameters of the cell are given in Table I.

Table 1: CRLH unit cell parameters

Variables	Length (mm)
Interdigital finger length	10.2
Interdigital finger width	0.3
Gap between fingers	0.3
Stub length	10.9
Stub width	1
Via radius	0.25
Substrate thickness	1.57

The CRLH unit cell has an infinitesimal length$\left(p \ll \lambda_g\right)$, where λg is the guided wavelength. Thus the CRLH unit cell can be characterized by an equivalent circuit model based on LC parameters of a transmission line (figure 2). For the ideal case, the interdigital capacitor of CRLH unit cell is synthesized by series capacitance C_L and series inductance L_R, while the stub inductor is a shunt inductor L_L and a capacitance C_R.

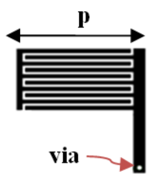


Figure 1: CRLH unit cell

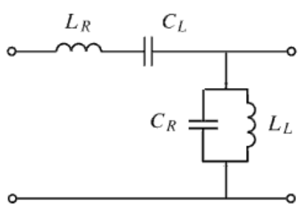


Figure2: Equivalent circuit model of a CRLH unit cell

Open waveguides or transmission lines act as a leaky-wave antenna if their dispersion diagram crosses the radiation cone. Radiation cone is the region delimited by the condition $-k_0 < \beta < k_0$, (k_0 is the free space propagation constant). The propagation constant β and the attenuation constant α of a CRLH structure, of a total length $\boldsymbol{l}$, can be calculated from its transmission coefficient.

$$S_{21} = |S_{21}|e^{j.arg(S_{21})} = e^{-\alpha l}e^{-j\beta l} \qquad (1)$$

The scan angle θ_0 of a CRLH leaky-wave antenna can be determined by the propagation constant β [2]:

$$\theta_0 = \sin^{-1}\left[\frac{\beta_{CRLH}(f)}{k_0(f)}\right] \qquad (2)$$

Interdigital capacitors used in the CRLH unit cell implementation require fine meshing in the electromagnetic simulation and then very long simulation duration. In order to describe the leaky-wave behavior of the CRLH structure, an analytical model can be advantageously used.

2.1. Analytical approach for 1D CRLH leaky-wave antenna

To describe radiation patterns of the CRLH based antenna, an estimation of the electromagnetic properties of the linear CRLH radiating structure is needed. Different analytical approaches have been proposed to predict radiation patterns of CRLH leaky-wave antenna [3], [4]. The first one, applied here, uses an array factor approach. Because the CRLH leaky-wave antenna is a 1D periodic structure so it can be described as an antenna array where each antenna is represented by a CRLH unit cell. The spacing between antennas of the array is the length of the CRLH unit cell.

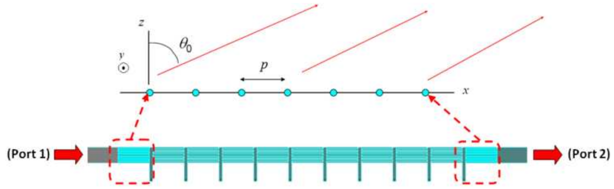


Figure 3: Array approach of a CRLH leaky-wave antenna

Due to its small length, the radiation pattern of the CRLH unit cell seems to be quasi isotropic. Nevertheless, electromagnetic simulations show that radiation patterns of the CRLH unit cell R_{UC} exhibit a maximum in broadside and minima at θ=±90°. This electromagnetic behavior looks like cos (θ). In order to improve the evaluation of the minima of the radiation patterns, a 0.5 factor is added in the description of the CRLH unit cell radiation patterns:

$$R_{UC}(\theta,\varphi) = \cos(0.5 * \theta) \qquad (3)$$

Each CRLH unit cell is fed with an amplitude function $\mathbf{I_m}$ and a phase function $\boldsymbol{\chi_m}$.

$$I_m = I_0 e^{-\alpha m p} \qquad (4)$$

$$\chi_m = -mk_0 p \sin\theta_0 = -m\beta_{CRLH} p \qquad (5)$$

p is the CRLH unit cell length
α is the leakage factor of the CRLH structure
β is the propagation constant of the CRLH structure
m is the location of each CRLH unit cell in the linear CRLH radiating structure
M is the total number of CRLH unit cells of the linear CRLH radiating structure

The array factor of the CRLH radiating structure is:

$$AF(\theta,\varphi) = \sum_{m=0}^{M-1} I_m e^{jm(k_0 p \sin\theta\cos\varphi + \chi)} \qquad (6)$$

$$AF(\theta,\varphi) = \sum_{m=0}^{M-1} I_m e^{jm(k_0 p \sin\theta\cos\varphi - k_0 p \sin\theta_0)} \qquad (7)$$

The radiation pattern of the CRLH leaky-wave antenna can then be estimated by this analytical expression:

$$R(\theta,\varphi) = R_{UC}(\theta,\varphi) * AF(\theta,\varphi) \qquad (8)$$

$$R(\theta,\varphi) = \cos(0.5 * \theta) * \sum_{m=0}^{M-1} I_m e^{jm(k_0 p \sin\theta\cos\varphi - k_0 p \sin\theta_0)} \qquad (9)$$

2.2. Validation of the 1D analytical approach

A CRLH leaky-wave antenna composed by 16 cells (Fig.4) is simulated with the TLM (Transmission Line Matrix) solver of CST Microwave Studio®. The CRLH unit cell has identical geometrical parameters than those presented on Table 1. However via shorted stubs are alternated in order to reduce cross-polarization level in radiation patterns [5].

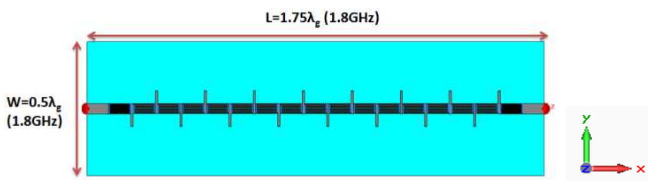


Figure 4: CRLH leaky-wave antenna with 16 cells

Full-wave simulation gives S-parameters of the CRLH antenna. Dispersion diagram is obtained from the phase of the transmission coefficient (Eq.1).

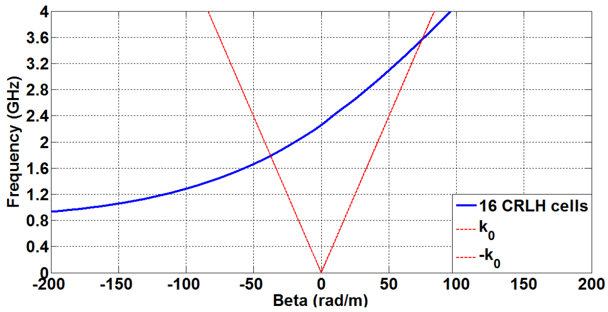


Figure 5: Dispersion diagram of the 16 cells CRLH leaky-wave antenna

Figure 5 shows that the CRLH leaky-wave bandwidth appears to be from 1.75 GHz to 3.6 GHz, with a zeroth order resonance (β=0) at 2.3 GHz.

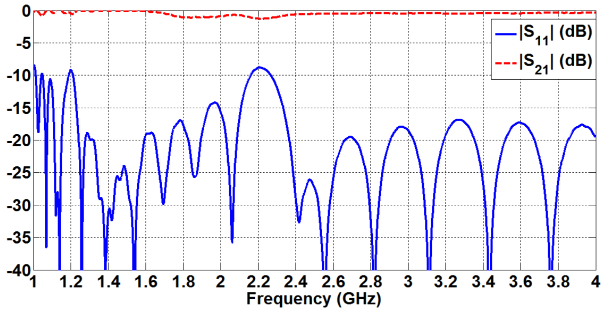
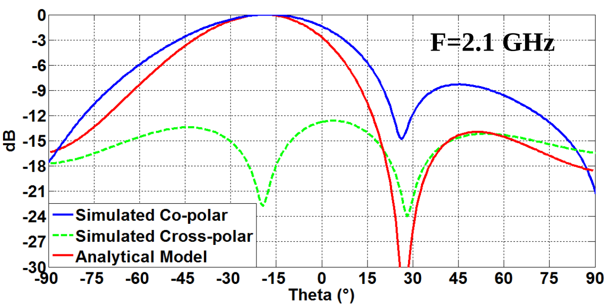


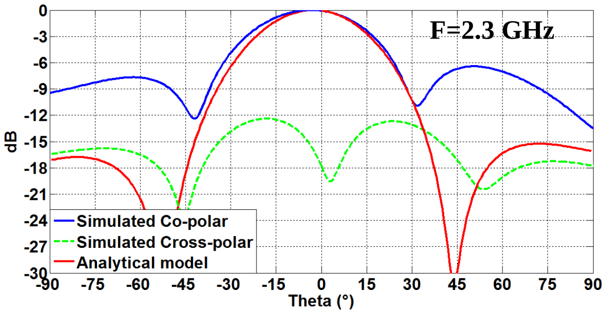
Figure 6: Simulated reflection and transmission coefficients of the 16 cells CRLH leaky-wave antenna

The return loss and the transmission coefficient of the CRLH antenna are presented in figure 6. The return loss of the CRLH antenna is less than -15dB from 1GHz to 4 GHz. However, close to the zeroth order resonance (2.3 GHz) from 2.2 GHz to 2.4 GHz, it goes around -7dB. This mismatching is due to the high variation of the input impedance around the zeroth order resonance. The transmission coefficient varies from -1 dB to -0.6 dB in the leaky-wave band (1.75-3.6 GHz). The radiation efficiency of the CRLH antenna can be calculated from S-parameters:

$$Radiation\ efficiency(\%) = 1 - |S_{11}|^2 - |S_{21}|^2 \quad (10)$$

Then S-parameters show that the radiation efficiency of the 16 cells CRLH leaky-wave antenna is approximately 20%.
In figure 7and figure 8, simulated radiation patterns are compared to those obtained with the 1D analytical model described above.
Radiation patterns synthesized by the analytical model illustrate well the scanning capability of the CRLH leaky-wave antenna. The direction of the main lobe and the HPBW are well estimated by the analytical model for the three frequencies presented in figure 7.Simulations exhibit in the (xOz) plane at least a difference of -13dB between co-polarization and cross-polarization. The HPBW is also well approximated in the (yOz) plane, but there are some level differences. Indeed the analytical model supposes that the CRLH leaky-wave antenna accepted power is all radiated on the polarization plane (xOz).





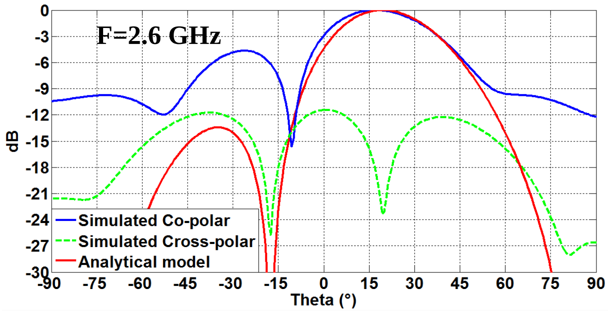
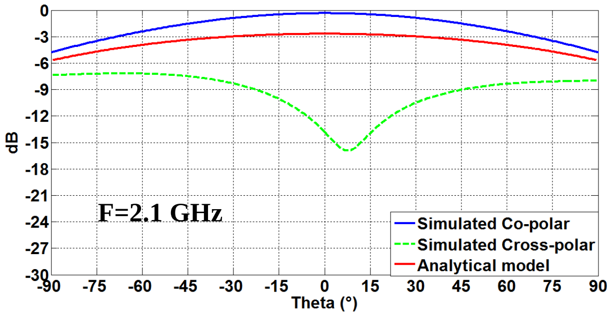
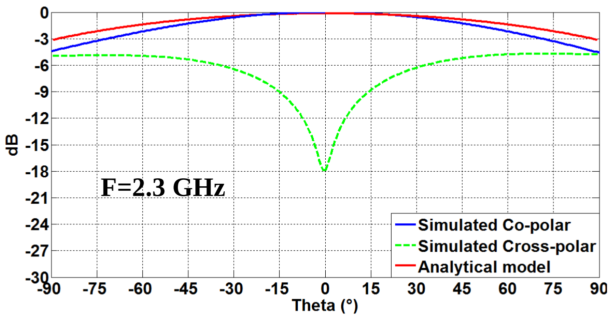


Figure 7: Radiation patterns of the 16 cells CRLH leaky-wave antenna in (xOz) plane (φ=0°)





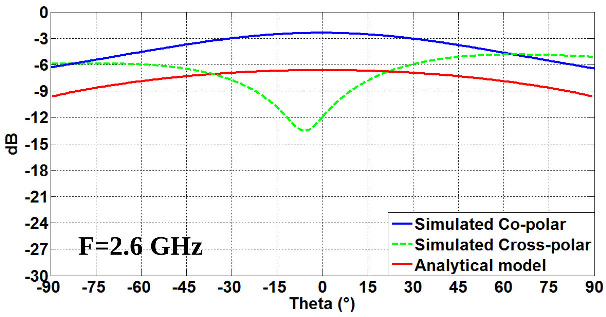


Figure 8: Radiation patterns of the 16 cells CRLH leaky-wave antenna in (yOz) plane (φ=90°)

3. CRLH based antennas

CRLH radiating structures can be used to design innovate antennas. The objective is to use the model based on electromagnetic properties of CRLH radiating structures to predict radiation patterns of the whole antenna.
The proposed CRLH based antenna has a shape of a meander line in which two CRLH structures are horizontal branches connected with microstrip bends as shown in figure 9. The linear CRLH leaky-wave antenna, composed of M cells, presented above is used to construct this CRLH based antenna. For this antenna, a 2D analytical model is developed.

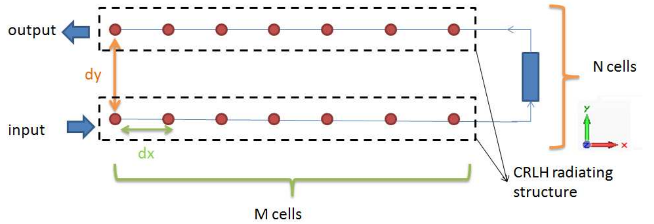


Figure 9: Array approach of the CRLH based meander antenna

3.1. Prediction of the S-parameters of the CRLH based meander antenna

The ABCD matrix of the complete antenna is calculated by cascading ABCD matrix of each branch of the meander antenna.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\substack{meander \\ antenna}} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{CRLH_1} * \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\mu strip} * \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{CRLH_2} \quad (11)$$

S-parameters of the meander antenna are then obtained from the ABCD matrix [6] by using equations (11) to (14), where Z_0 is the characteristic impedance of the CRLH structure.

$$A = \frac{(1+S_{11})(1-S_{22})+(S_{12}S_{21})}{2*S_{21}} \quad (12)$$

$$B = Z_0 . \frac{(1+S_{11})(1+S_{22})-(S_{12}S_{21})}{2*S_{21}} \quad (13)$$

$$C = \frac{1}{Z_0} . \frac{(1-S_{11})(1-S_{22})-(S_{12}S_{21})}{2*S_{21}} \quad (14)$$

$$D = \frac{(1-S_{11})(1+S_{22})+(S_{12}S_{21})}{2*S_{21}} \quad (15)$$

The meander antenna is made of two identical CRLH radiating structures, so only S-parameters of the CRLH radiating structure and microstrip bends length are needed to calculate S-parameters of the whole antenna.

3.2. Prediction of radiation patterns of the CRLH based meander antenna with an analytical model

The CRLH based meander antenna is considered as a 2D array antenna, where each antenna is a CRLH unit cell. Each unit cell is identified by its location on (Ox) and (Oy) axis respectively by *dx* and *dy*.

For the 1st CRLH radiating structure, each *m* CRLH unit cell is fed with an amplitude function I_m^1 and phase amplitude χ_m^1. χ_m^1 is, for each CRLH unit cell of the 1st line, the phase delay introduced by previous CRLH unit cells.

$$I_m^1 = I_0 e^{-\alpha m dx} \quad \text{and} \quad \chi_m^1 = -(\beta * m * dx) \quad (16)$$

So the array factor of the first CRLH radiating structure is:

$$AF_{CRLH1}(\theta,\varphi) = \sum_{m=0}^{M-1} I_m^1 e^{j\left((k_0 * m * dx * \sin\theta * \cos\varphi) + \chi_m^1\right)} \quad (17)$$

For the 2nd CRLH line, each *m* CRLH unit cell is fed with an amplitude function I_m^2 and a phase function χ_m^2. χ_m^2 is the sum of the total phase delay introduced by the 1st CRLH line, the phase delay set by CRLH unit cells preceding this considered cell and the phase delay introduced by microstrip bends connecting the 1st and 2nd CRLH lines.

$$I_m^2 = I_0 e^{-\alpha * \left(((M-1)*dx) + m*dx\right)} \quad (18)$$

$$\chi_m^2 = -\left((\beta * (M-1) * dx) + (\beta * (M-1-m) * dx)\right) + \delta \quad (19)$$

δ is the phase delay introduced by microstrip bends connecting the two CRLH radiating structures.
Then the array factor of the 2nd CRLH radiating structure is:

$$AF_{CRLH2}(\theta,\varphi) = \sum_{m=0}^{M-1} I_m^2 e^{j\left((k_0 * (m * dx * \sin\theta * \cos\varphi) + (dy * \sin\theta * \sin\varphi)) + \chi_m^2\right)} \quad (20)$$

The radiation pattern R(θ,φ) of the meander antenna is then obtained by a summation of radiation patterns of each CRLH radiating structure which are determined by the product of the single CRLH unit cell $R_{UC}(\theta,\varphi)$ and its array factor $AF_{CRLH1,2}$ (Equ.21). CRLH unit cells that are in the same CRLH radiating structure have the same polarization vector (p_1=1). However CRLH unit cells of the 2nd CRLH structure are polarized oppositely to those of the 1st one.

$$R(\theta,\varphi) = R_{UC}(\theta,\varphi) * p_1 * \left(AF_{CRLH1}(\theta,\varphi) - AF_{CRLH2}(\theta,\varphi)\right) \quad (21)$$

3.3. Validation of the analytical model

To validate prediction of the radiation patterns obtained with the analytical model presented above, a CRLH based meander antenna is simulated. The meander antenna is realized with two linear CRLH radiating structures. Each CRLH structure has 15 cells.

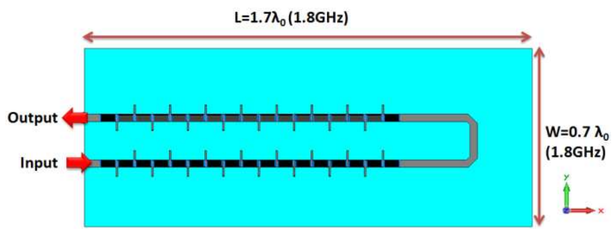


Figure 10: CRLH based meander antenna

Full-wave simulation of the CRLH based meander antenna is performed with the TLM (Transmission Line Matrix) solver of CST Microwave Studio®.
S-parameters obtained with the model by cascading chain matrixes of each element of the meander antenna (Eq.11) are compared with simulated S-parameters in figure 11.

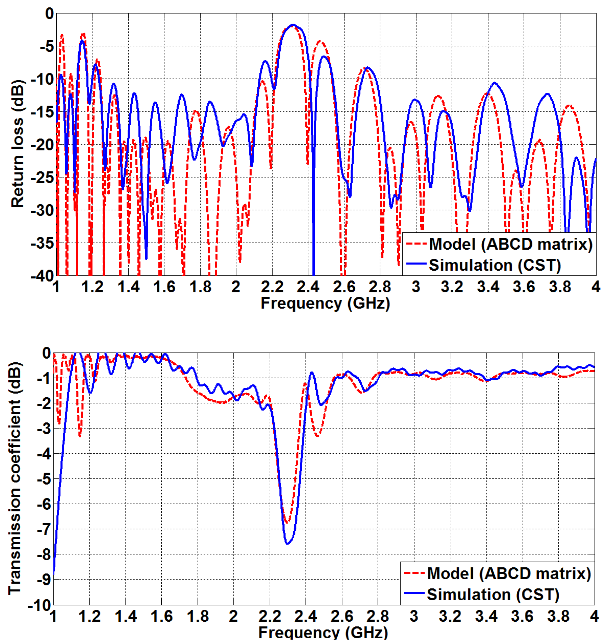


Figure 11: Validation of S-parameters prediction

Fig.11 exhibits a frequency shift of about 80MHz between simulation and model. The maximum difference between simulated and analytic transmission coefficients is about 0.7dB, while it is 5dB between simulated and analytic reflection coefficients. These dissimilarities can be explained by the cascade of the chain matrix of the different CRLH unit cells (in the analytical model), which does not take into account the coupling between CRLH unit cells.
The CRLH based meander antenna exhibits a good impedance matching ($|S_{11}|$<-10dB) from 1.5 GHz to 4 GHz, except around the zeroth order resonance like for the CRLH linear 16 cells presented above.
The radiation efficiency of the CRLH based antenna is calculated from S-parameters (Eq.10). Simulated and analytical S-parameters show that the maximum radiation efficiency of the CRLH based meander antenna is 25% at f=1.9 GHz. This radiation efficiency can be improved with a higher number of CRLH radiating structures.
Radiation patterns obtained with the analytical model are compared to the simulated results (Fig.12 and Fig.13). To validate the results obtained with the analytical model, three frequencies are chosen: f=1.9GHz (β<0), f=2.4GHz (β=0) and f=2.6 GHz (β>0). HPBW of the different radiation patterns are well estimated by the analytical model. We note a good agreement between simulation and analytical model especially in the (xOz) plane (φ=0°). Indeed in the model it is supposed that the antenna accepted power is all radiated in the (xOz) plane. That is why there are some discrepancies between simulation and model in the (yOz) plane.

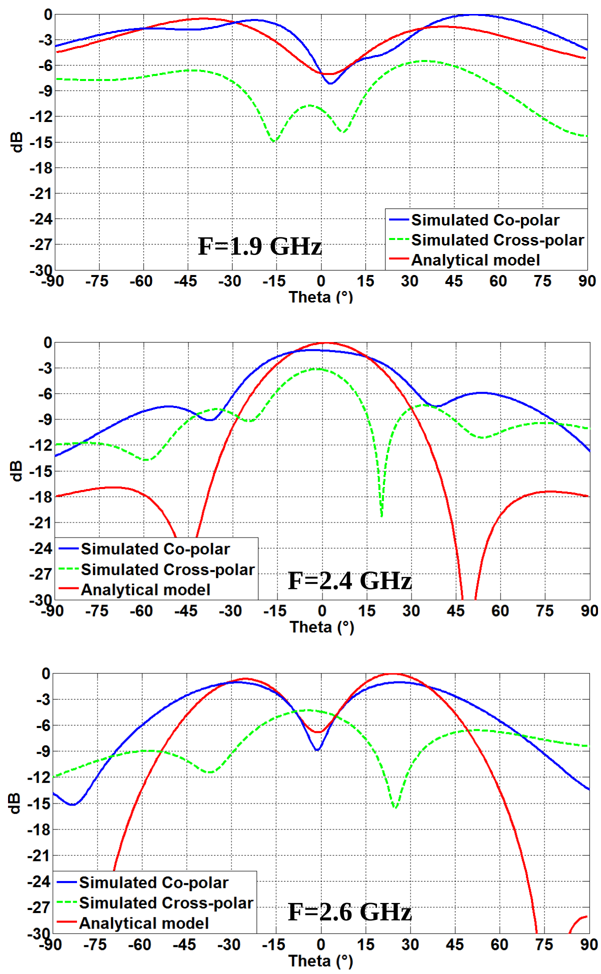
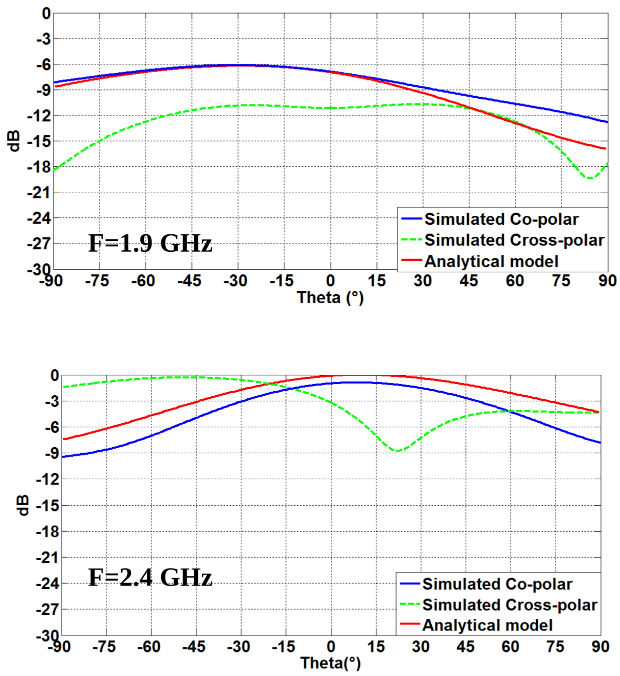


Figure 12: Radiation patterns of the CRLH based meander antenna in (xOz) plane (φ=0°)



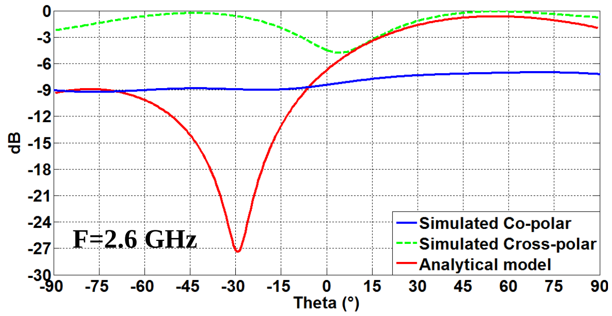


Figure 13: Radiation patterns of the CRLH based meander antenna in (yOz) plane (φ=90°)

The CRLH based meander antenna exhibits interesting behaviors. It has radiation patterns with large HPBW at the limit frequencies of the leaky wave bandwidth and when we approach the zeroth order resonance the HPBW of the main lobe is reduced. By adjusting appropriately the microstrip bends connecting the CRLH radiating structures of the meander antenna, radiation patterns of the whole antenna can be altered. We can obtain a main lobe more or less directive in broadside or another direction. In the other hand we can obtain radiation patterns with a high HPBW >±60°, and a maximum level at broadside more than -3dB. This last case would be very interesting compared to classical antenna arrays which provide a maximal HPBW of ±60°.

4. Conclusions

CRLH based antennas are designed by connecting CRLH radiating structures. An analytical model has been proposed to predict quickly their performance whatever the geometry of the complete antenna is. This model uses electromagnetic properties of the single CRLH radiating structure. Results obtained with full 3D electromagnetic simulation software prove the validity of the analytical model. This model is very useful because it provides rapidly performances of the CRLH based antenna while full 3D simulation software take long simulation duration (more than 3 days for a CRLH based meander antenna of 15 cells instead of 1 minute with the model). Thus the model can now be used to find the adequate geometry of the CRLH based antenna in order to have a desired radiation pattern.

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