METALLIC GRADED PHOTONIC CRYSTALS FOR GRADED INDEX LENS

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Abstract

We have designed a flat graded index lens made from a metallic graded 2D photonic crystal. The gradient of index has been obtained by varying the filling factor of a flat slab of photonic crystal in the direction perpendicular to that of the propagation of the electromagnetic field. This gradient has been designed in such a way that the flat slab focuses a plane wave. With applications in the microwave range in view, we considered a photonic crystal which consists of copper strips.

1. Introduction

The relation of dispersion of photonic cristals $\omega = \omega(\mathbf{k})$ is as a band structure, so that there are bandgaps in which the electromagnetic field cannot propagate. Apart from the bandgaps of photonic crystals, that is, in the photonic bands, the propagation of the electromagnetic field is governed by the shape of these bands [1]. Besides, graded photonic crystals (GPC) have been demonstrated to enhance the ability of photonic crystals to control the light propagation [2]. GPC are obtained by gradual modications of photonic crystal parameters, such as the lattice period, the filling factor or the dielectric constant. Several phenomena involving GPC have already been demonstrated such as light bending, quasi-transparency or focusing [3, 4, 5, 6]. In this communication, we report on the design of a GPC slab whose filling factor was varied. The gradient of the filling factor results in a gradient of the index of refraction, which allows the slab to focus a plane wave. The GPC was designed so as to it behaves as an homogeneous isotropic material. We consider a metallic photonic crystal for applications in microwave domain, such as antenna applications [7, 8, 9, 10], because in this domain of frequency, losses are very low [11].

2. Design

On one hand, gradient-index optics finds many applications in imagery and in telecommunications and has been quite studied for a long time (see reference [12] for a review). On the other hand, photonic crystals permit to control the flow of the electromagnetic field *via* the shape of their photonic bands which rely the group velocity $\mathbf{v_g}$ to the wave vector

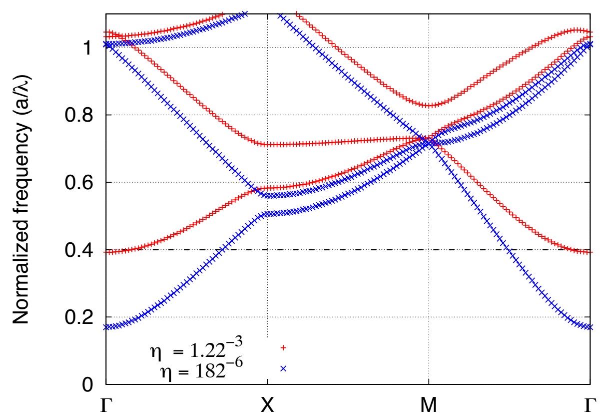

Figure 1: Band structures of two 2D metallic photonic crystals made of copper strips arranged in a square lattice with lattice constant $a = 12\,\mathrm{mm}$. The incident electric field is parallel to the axis of the strips (TM mode). In the first photonic band, at the frequency $\lambda/a = 0.4$, the IFC are circular. The width of the strips of each photonic crystal is constant. The blue and the red curves correspond to the smallest width and to the greatest width of the strips, respectively.

$\mathbf{k}$, according to the relation [1]

$$\mathbf{v_g} = \nabla_{\mathbf{k}}\, \omega(\mathbf{k})\,.$$

Thereby, the wave vector $\mathbf{k}$ is perpendicular to the isofrequency curves (IFC). These represent the relation of dispersion at a given frequency.

On their part, GPC rely on a small variation of one of the parameters over one period of the lattice of the photonic crystal. This small variation gradually modifies the dispersion properties. Consequently, engineering the IFC allows the control of the direction of the wave vector $\mathbf{k}$. IFC may have a great variety of shapes. If these are circular shapes, the photonic crystal can be assumed as a homogeneous isotropic medium. Moreover, at the interface of the photonic crystal with vacuum, the tangential component of the wave vector $\mathbf{k}$ is continuous. These are the key points to

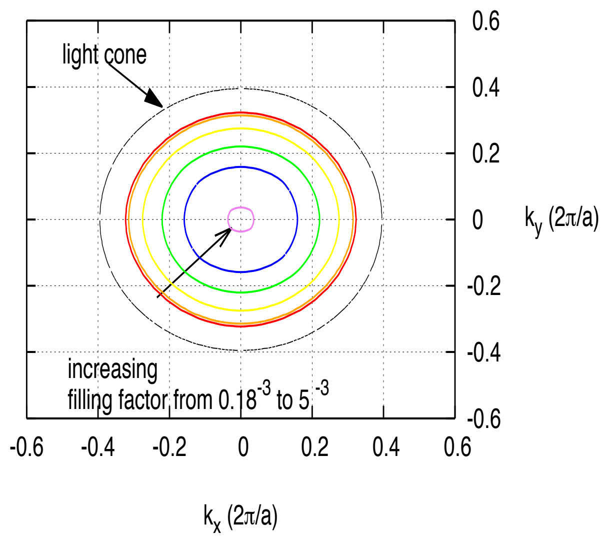

Figure 2: Six iso-frequency curves and that of the vacuum at $\lambda/a = 0.4$, the latter being referred to as the light cone. Each of these curves is circular and corresponds to width of the constitutive of the GPC. The filling factor varies from $0.18\,10^{-3}$ (red curve) to $5\,10^{-3}$ (purple curve).

design a graded index lens from a graded photonic crystal.

For applications in the X band frequency range, we considered a 2D photonic crystal made of metallic strips periodically arranged in a square lattice whose side is a. The strips are onto a dielectric substrate. Indeed, in the microwave range, metals can be considered as ideal metals, because ohmic losses are very low [11]. Thus, the elementary cell of the considered photonic crystal consists of a 12 mm wide square within which a 35 μm thick strip is centered. The filling factor η is given by

$$\eta = \frac{t\,w}{a^2}$$

So as to design the GPC, the filling factor η has been varied by varying the width w of the strips. We calculated band structures and IFC for several values of the filling factor η *via* a "home made" Finite Difference Time Domain (FDTD) code source which describes the unit cell with Periodic Boundaries Conditions [13]. The polarization of the incident wave was transverse magnetic (TM), the electric field being parallel to the axis. The conductivity of the copper strips is taken into account ($\sigma = 5.9\,10^7 S/m$) in the simulations. As the thickness of the strips is very small against their width, this would have necessitated a very fine mesh size and consequently very long numerical calculations. Consequently, we carried out the FDTD simulations using a subgridding scheme based on mesh nesting [14]. Thereby, the dielectric substrate is not simulated.

In the first photonic band of square metallic lattices, the IFC may be circular. Two band structures, which give rise to circular IFC, are reported in Fig. 1, that of the smallest width of the strips and that of the greatest width of the strips. The working frequency is $\lambda/a = 0.4$. The corresponding IFC for six values of the filling factor η and that of the vacuum are shown on Fig. 2. When the IFC are circular, the effective index n_{eff} can be calculated from the ratio of the radius of the IFC to that of the relation of dispersion of the vacuum. This latter is generally called the "light cone". Thus, we deduced a "calibration curve", that is, the variation of the effective index n_{eff} with the filling factor η, $n_{eff} = n_{eff}(\eta)$. The former decreases as the latter increases.

Then, we considered the graded index lens, that is, a slab of homogeneous medium, whose index of refraction is modified from center towards the edges. Graded index lens acts as a phase compensator (see Fig. 3), that is, all optical paths across the slab are equal [15]. The shape of the positive index of refraction is given by [15]

$$n(r) = n(0) - \frac{\sqrt{r^2 + f^2} - f}{d},$$

where f is the focal length of the lens, d is its thickness, r is the radial distance from the optical axis and $n(0) = 1$ is the index of refraction along the optical axis of the lens. The focal length and the thickness were chosen $f = 15\,\text{cm}$ and $d = 6\,\text{cm}$, respectively. Thanks to the previously deduced relation between the effective index n_{eff} and the filling factor η ($n_{eff} = n_{eff}(\eta)$), we extrapolated the different widths of the constitutive strips in order to design the required gradient index $n(r)$. The width of the strips increases from the optical axis towards the edges. As the width w of the strips increases, the effective index n_{eff} decreases. A sketch of the whole device, made of printed circuit board (PCB), is reported in Fig. 4.

3. Simulations

Then, we simulated the whole device *via* a "home made" FDTD code source which involves Perfectly Matched Layer boundaries conditions and the Total Field/Scattered Field method [13]. It consisted of five layers of copper strips. Simulations were carried out at 10 GHz (normalized frequency $a/\lambda = 0.4$), firstly with a plane wave incident on the lens. The results of the simulations are reported in Fig. 5 and the map of the mean value of the square of the electric field ($\overline{E_z^2}$) highlights that the simulated focal length is around $f = 15\,\text{cm}$. Actually, the thickness d of the slab is not negligible compared to the focal length f, so that the slab cannot be assumed as a thin lens. The corresponding shape of the electric field in the focal plane is reported in Fig. 6. Full width at half maximum (FWHM) is 2.35 cm, so that FWHM $= 0.78\lambda$. It can be seen that the outer secondary maximum are high. Edge effects have been attributed to this. Indeed, it can be noticed in Fig. 5 that the electric field is not null on the edges of the domain of simulation. The discretization of the constitutive strips

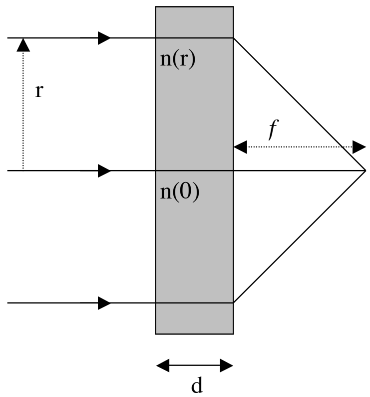

Figure 3: Gradient index lens which behaves as a phase compensator, that is, all the incident parallel rays converge towards the focal point with the same optical path. The index of refraction $n(r)$ is varied symmetrically and perpendicularly to the axis of the lens, from $r = 0$ (optical axis of the lens) towards the edges. f is the focal length of the lens and d is its thickness.

by a square mesh also brings about discrepancy in the simulations.

Secondly, so as to confirm our device really works as a graded index lens, we carried out further simulations with a punctual source located at the focal point. The results of the simulations are reported in Fig. 7 and it can be noticed that the designed GPC actually transforms a cylindrical incident wave into a plane wave. Both cases of simulations showed that the slab acts as a convex lens which focuses an incident plane wave and which transforms a cylindrical wave into a plane wave. Besides, this devices comprised only a few layers, so that it proves that GPC have the ability to efficiently control the propagation of light.

4. Conclusions

We designed a metallic graded photonic crystal slab so that its gradient of filling factor reproduces a gradient of index resulting in a given focal length. Then, we demonstrated firstly, that this device focuses a plane wave at the requisite focal length and secondly, that it transforms a cylindrical wave issued from a punctual source located at the focal point into a plane wave. Therefore, the graded photonic crystal slab behaves like a convex lens. Although this device applies to the microwave frequency range, GPC have the potential to optical components and would efficiently apply to integrated optics devices. Moreover, the engineering of the IFC should permit to design of gradient of negative index and that way, the design gradient index lens with sub-wavelength resolution. Therefore, GPC can find a great variety of applications from the microwave range to the optical domain and may be easily used in various integrated photonic devices.

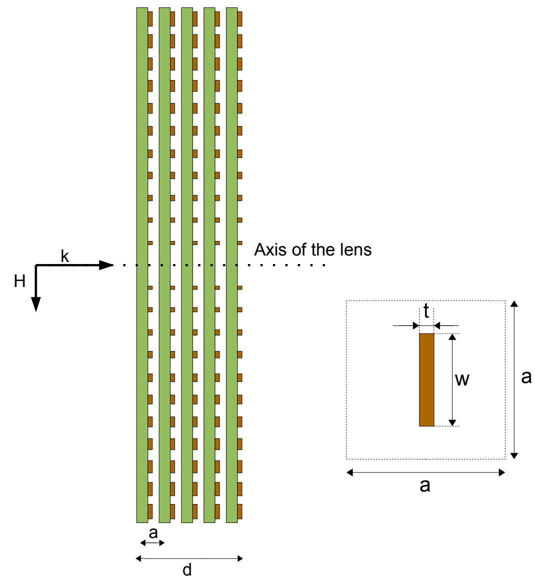

Figure 4: Sketch of the graded 2D photonic crystal slab made of metallic strips onto a dielectric substrate. The width w of the strips increases from the axis of the slab towards the edges, whereas their thickness t is constant to 35 μm. a is the period lattice which is constant. The direction of the wave vector $\mathbf{k}$ is shown and is perpendicular to the gradient. There is no strip along the axis of the lens since $n(0) = 1$. Inset : The elementary cell is a square of side a within which a 35 μm thick strip is centered.

References

[1] M. Notomi, "Theory of light propagation in strongly modulated photonic crystals: Refractionlike behavior in the vicinity of the photonic band gap," *Phys. Rev. B*, vol. 62, no. 16, pp. 10 696–10 705, Oct 2000.

[2] E. Centeno and D. Cassagne, "Graded photonic crystals," *Optics Letters*, vol. 30, no. 17, pp. 2278–2280, SEP 1 2005.

[3] E. Akmansoy, E. Centeno, K. Vynck, D. Cassagne, and J.-M. Lourtioz, "Graded photonic crystals curve the flow of light: An experimental demonstration by the mirage effect," *Applied Physics Letters*, vol. 92, no. 13, MAR 31 2008.

[4] E. Centeno, E. Akmansoy, K. Vynck, D. Cassagne, and J.-M. Lourtioz, "Light bending and quasi-transparency in metallic graded photonic crystals," *Photonics And Nanostructures-Fundamentals And Applications*, vol. 8, no. 2, pp. 120–124, MAY 2010.

[5] H. Kurt, E. Colak, O. Cakmak, H. Caglayan, and E. Ozbay, "The focusing effect of graded index photonic crystals," *Applied Physics Letters*, vol. 93, no. 17, p. 171108, 2008.

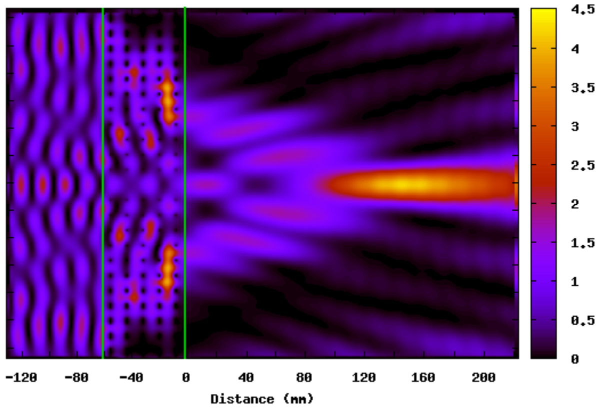

Figure 5: Map of the mean value of the square of the electric field ($\overline{E_z^2}$) of an incident TM plane wave on a graded index lens of focal length $f = 15\,cm$. The lens consists of a graded photonic crystal made of five layers of copper strips whose width is varying perpendicularly to the direction of propagation. The two vertical green lines delimit the lens.

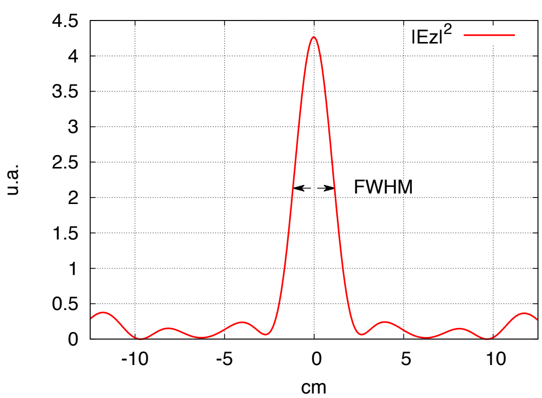

Figure 6: Shape of the mean value of the square of the electric field ($\overline{E_z^2}$) in the focal plane extracted from Fig. 5. FWHM $= 0.78\lambda$.

[6] H. Kurt and D. S. Citrin, "Graded index photonic crystals," *Opt. Express*, vol. 15, no. 3, pp. 1240–1253, 2007.

[7] N. Guerin, S. Enoch, G. Tayeb, P. Sabouroux, P. Vincent, and H. Legay, "A metallic fabry-perot directive antenna," *Antennas and Propagation, IEEE Transactions on*, vol. 54, no. 1, pp. 220 – 224, jan. 2006.

[8] H. Boutayeb, T. Denidni, K. Mahdjoubi, A. Tarot, A. Sebak, and L. Talbi, "Analysis and design of a cylindrical EBG-based directive antenna," *IEEE

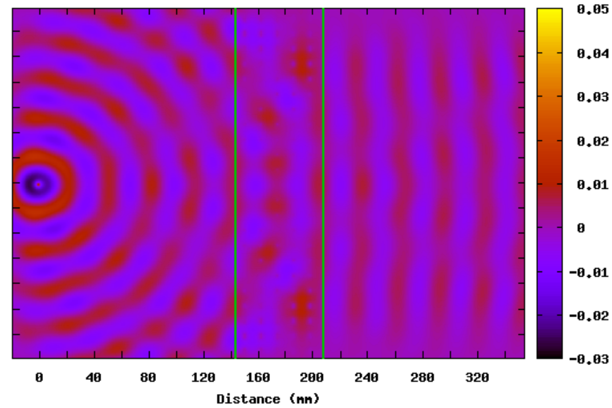

Figure 7: Map of the instantaneous electric field E_z of a punctual TM source located at the focal point and illuminating the graded index lens. The two vertical black lines delimit the lens.

Transactions On Antennas And Propagation, vol. 54, no. 1, pp. 211–219, 2006.

[9] P. Ikonen, C. Simovski, and S. Tretyakov, "Compact directive antennas with a wire-medium artificial lens," *Microwave and Optical Technology Letters*, vol. 43, no. 6, pp. 467–469, 2004.

[10] A. de Lustrac, F. Gadot, E. Akmansoy, and T. Brillat, "High-directivity planar antenna using controllable photonic bandgap material at microwave frequencies," *Applied Physics Letters*, vol. 78, no. 26, pp. 4196–4198, 2001.

[11] J. Lourtioz, *Photonic crystals: towards nanoscale photonic devices*. Springer, 2005.

[12] D. T. Moore, "Gradient-index optics: a review," *Appl. Opt.*, vol. 19, no. 7, pp. 1035–1038, 1980.

[13] A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method, 2nd ed.* Artech House Publishers, 2005.

[14] S. Chaillou, J. Wiart, and W. Tabbara, "A subgridding scheme based on mesh nesting for the fdtd method," *Microwave and Optical Technology Letters*, vol. 22, no. 3, pp. 211–214, 1999.

[15] A. O. Pinchuk and G. C. Schatz, "Metamaterials with gradient negative index of refraction," *J. Opt. Soc. Am. A*, vol. 24, no. 10, pp. A39–A44, 2007.