Plasmon beams interaction at interface between metal and dielectric with saturable Kerr nonlinearity

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Abstract

We present a novel theory of surface plasmon polariton interaction on the surface of dielectric with saturable Kerr nonlinearity. The effect of the total internal reflection of a weak signal plasmon beam from a high-power reference beam is discussed. Both ray and wave theories are used to describe signal propagation. The effect of the signal tunneling through the narrow inhomogeneity induced by the reference beam is considered.

1. Introduction

Surface plasmon polariton (SPP) is electromagnetic wave propagating along the interface of media with different signs of permittivity or permeability values. Recently besides widely known SPP on metal–dielectric interfaces [1, 2] also SPP on left-handed [3, 4] and exotic metamaterial interfaces [5] are studied.

Optical SPP is localized in a very thin layer of less than 1 μm and exponentially decays in the direction transverse to the interface. This unique feature can be used both in nonlinear photonics due to very high energy concentration and in compact plasmonic devices. Therefore the methods of SPP control are of prime interest. Nowadays several methods of SPP management by nanoparticles or heterostructure design are proposed [6, 7].

We focus our attention on the possibility of light-by-light control that has a great importance for nonlinear optics and photonics [8, 9]. The properties or direction of a signal laser beam are controlled by a reference beam. One of the promising approaches is to combine methods of nonlinear photonics with the recent success of plasmonics [10, 11] in order to engineer high-speed and compact devices [12]. In this work the possibility of signal surface plasmon propagation direction management with the high-power reference plasmon via the phenomenon of total internal reflection is demonstrated.

The total internal reflection is a well-known optical phenomenon that takes place if the light falls on a boundary of a less dense medium. However the inhomogeneity of the refractive index induced in the nonlinear media by a high-power laser beam can act like such boundary, too, causing the opacity of the reference beam region and the total reflection of the signal beam. The phenomenon of the nonlinear total internal reflection of the bulk laser beams was considered in Ref. [13, 14]. This work is devoted to the plasmon interaction with the inhomogeneity of the dielectric permittivity induced by the high-power reference plasmon in the dielectric with saturable Kerr nonlinearity.

2. Theory of SPP beam propagation on surface of nonlinear dielectric

We consider an interaction of two plasmon beams at the interface between the dielectric with saturable Kerr nonlinearity (i.e. photorefractive crystal of lithium niobate) and the noble metal (i.e. silver or gold). The following differential equation describes the monochromatic reference plasmon propagation along the interface:

$$\Delta\vec{E} - \text{grad div}\vec{E} + \varepsilon_j k_0^2 \vec{E} = 0, \tag{1}$$

where index $j = d, m$ corresponds to metal or dielectric, $k_0 = \omega/c$ is the wave number in vacuum. The geometry of the problem is selected to be as follows: Oz axis is perpendicular to the interface, the dielectric and metal occupy half-spaces $z > 0$ and $z < 0$ correspondingly. The pump plasmon beam propagates along Ox axis, the signal beam propagates at a small angle φ to it.

Under the influence of strong electromagnetic field the dielectric permittivity ε_d changes:

$$\varepsilon_d\left(|\vec{E}|^2\right) = \varepsilon_L + \varepsilon_{NL}\left(|\vec{E}|^2\right) = \varepsilon_L + \chi\frac{|\vec{E}|^2}{\alpha + |\vec{E}|^2}, \tag{2}$$

where ε_L is the linear part of dielectric permittivity, and nonlinear part ε_{NL} is determined by the field intensity with the coefficients χ describing nonlinearity and α corresponding to saturation intensity value. This model is usually used for the description of nonlinear effects in the photorefractive crystals (see, for example, [14]).

For the solution of the Eq. (1) we use the slowly varying amplitude method adapted for the paraxial plasmonic beams that was described in Ref. [15] in detail. We seek the solution of Eq. (1) in the following form:

$$\vec{E} = \left[A(x,y)E_{x0}(z)\vec{e}_x - iB(x,y)E_{z0}(z)\vec{e}_z\right]e^{-i\beta x} \tag{3}$$

where A, B are the slowly varying amplitudes and

$$\vec{E_0} = \{E_{x0}, 0, E_{z0}\} \tag{4}$$

is the transversal profile obtained as a solution of linear equation (1):

$$E_{x0} = e^{-\gamma_j|z|}, \tag{5}$$

$$E_{z0} = \frac{\beta}{\gamma_j} sgn(z) e^{-\gamma_j |z|}. \quad (6)$$

The propagation constant β and the localization coefficients γ_j are determined as:

$$\beta = k_0 \sqrt{\frac{\varepsilon_m \varepsilon_L}{\varepsilon_m + \varepsilon_L}}, \quad (7)$$

$$\gamma_j = k_0 \sqrt{-\frac{\varepsilon_j^2}{\varepsilon_m + \varepsilon_L}}. \quad (8)$$

Note that in the more complicated case (e.g. on gyrotropic interface [16]) SPP has another profile and dispersion.

We assume that the plasmon profile and polarization remains nearly the same as in the linear problem ($A \approx B$, $\frac{\partial A}{\partial z} = 0$) and the beams are rather wide so that $|\frac{\partial A}{\partial x}| \ll \beta |A|$. Substituting (3) in Eq. (1), multiplying the equation projections onto Ox and Oz axis by E_{x0} and E_{z0} correspondingly, and integrating over transverse coordinate z the sum of the obtained equations we finally get

$$\frac{\partial^2 A}{\partial y^2} - 2i\beta\theta_1 \frac{\partial A}{\partial x} + i\theta_3 \Gamma A + \chi \theta_2 k_0^2 A \left[1 - \frac{\alpha\theta_3}{|A|^2} \ln\left(1 + \frac{|A|^2}{\alpha\theta_3}\right)\right] = 0, \quad (9)$$

Where Γ is the imaginary part of metal dielectric permittivity. The coefficients $\theta_{j=1,2,3}$ appear due to the integration that physically means averaging over transversal direction. These coefficients equal:

$$\theta_1 = 1 - \frac{\kappa}{1+\kappa^2}, \quad (10)$$

$$\theta_2 = \frac{1}{1+\kappa^2}, \quad (11)$$

$$\theta_3 = \frac{\kappa}{1+\kappa}, \quad (12)$$

where $\kappa = -\varepsilon_d/\varepsilon_m$. Notice that usually $|\varepsilon_m| \gg \varepsilon_d$ and $\kappa \ll 1$ so that the coefficient $\theta_1 \approx 1$ leading to the linear parabolic diffraction equation that was obtained in the linear case in our previous work [17]. The nonlinearity coefficient χ is multiplied by $\theta_2 \approx 1$ (in the extreme case of $\kappa \ll 1$) since the electromagnetic field is mainly concentrated in nonlinear dielectric. Surface plasmon decay is determined by the losses in metal and the last term multiplied by the coefficient θ_3 shows the relative amount of the electromagnetic field in metal. At the same time the saturation value α is re-normalized to θ_3 for convenience. The coefficient θ_3 actually accounts for field amplitude $|A(x,y)\vec{E}(0)|$ normalization.

Therefore surface plasmon propagation in the dielectric with saturable Kerr nonlinearity can be accurately described by the Eq. (9).

3. SPP beam propagation in the presence of SPP-induced inhomogeneity

We consider two SPP beams propagating at a small angle to each other. One of the SPP beams is high-power reference beam and another is weak signal beam. Therefore the influence of signal beam on the reference beam propagation can be neglected and only self-action, diffraction phenomena and decay taken into account:

$$\frac{\partial A_r(x,y)}{\partial x} + iD_r \frac{\partial^2 A_r(x,y)}{\partial y^2} + i\frac{\chi k_0^2}{2\beta_r\theta_{1r}} A_r(x,y) \\ i\frac{\theta_3 \Gamma}{2\beta_r\theta_{1r}} A \left[1 - \frac{\alpha\theta_{3r}}{|A_r|^2} \ln\left(1 + \frac{|A_r|^2}{\alpha\theta_{3r}}\right)\right] = 0. \quad (13)$$

In this equation index r refers to the reference beam, $D_r = \frac{1}{2\beta_r\theta_{1r}}$ is the diffraction coefficient.

The equation describing signal SPP (index s) propagation can be obtained in a quite similar way. We assume that self-action of the signal plasmon is weak and can be neglected while diffraction and the dielectric permittivity inhomogeneity induced by the pump should be taken into account:

$$\frac{\partial A_s(x,y)}{\partial x} + iD_s \frac{\partial^2 A_s(x,y)}{\partial y^2} + i\frac{\sigma(x,y)}{2\beta_s\theta_{1s}} A_s(x,y) = 0, \quad (14)$$

where the induced inhomogeneity:

$$\sigma(x,y) = \int_0^{+\infty} \frac{\kappa_s k_{0s}^3 \chi}{2\gamma_{2s}^2 \theta_s^2 \varepsilon_2} \frac{|A_r(x,y)|^2 |\vec{E}_{0r}(z)|^2 |\vec{E}_{0s}(z)|^2}{\alpha + |A_r(x,y)|^2 |\vec{E}_{0r}(z)|^2} dz. \quad (15)$$

is determined both by pump and signal plasmon transversal profiles. The sign of inhomogeneity coincides with the nonlinearity sign χ. The profile of the inhomogeneity due to the saturation of the nonlinearity does not exactly repeat the shape of the pump plasmon beam. As far as $\partial\sigma/\partial|A_r|^2 > 0$ the regions with higher plasmon intensity correspond to the larger inhomogeneity values.

Although the value of inhomogeneity σ can not be derived analytically it can be numerically calculated almost for every possible pump field distribution and pump and signal frequency ratios. For the intensities of the reference beam much smaller than the saturation intensity the induced inhomogeneity can be analytically derived as:

$$\sigma(x,y) = \theta_4 \chi \frac{|A_r(x,y)|^2}{\alpha}, \quad (16)$$

where the coefficient θ_4

$$\theta_4 = \frac{(1+\kappa_r)\gamma_{2s} k_0^2}{2\kappa_r \beta_s^2 \theta_{1s} (\gamma_{2s} + \gamma_{2r})}. \quad (17)$$

In this case the inhomogeneity profile repeats the profile of the reference beam.

Decay of signal surface plasmon can be described in a similar way to the pump plasmon. However the equation describing its propagation (14) is linear thus losses lead only to amplitude decay. Therefore we can consider lossless case for simplicity since the phenomenon described with this assumption would be exactly the same.

4. Ray theory of signal SPP propagation

Now we assume that the induced effective inhomogeneity $\sigma(x,y)$ is known and focus our attention on the possible regimes of signal plasmon beam propagation. For the simplification of our analysis we derive the equation for the signal plasmon trajectory. We use the eikonal method of ray theory that was described in [13] in a detail. The equation for the signal plasmon beam trajectory is:

$$\frac{d^2y}{dx^2} = \frac{\partial\sigma(x,y)}{\partial y}. \quad (18)$$

If we consider a rather small interaction region so that its sizes do not exceed the pump diffraction and nonlinear lengths we can assume that the reference plasmon amplitude does not change significantly in this region so that $A_r(x,y) \approx A_r(y)$. Therefore the inhomogeneity $\sigma = \sigma(y)$ and the solution of the Eq. (18) have the following form:

$$x = x_0 \pm \sqrt{(}2)\int\limits_{y_0}^{y} \frac{d\xi}{\sqrt{\sigma(\xi) - \sigma(y_0) + \varphi^2/2}}. \quad (19)$$

For the different parts of the trajectory the sign before the integral is different. Notice that the solution in defocusing media ($\sigma < 0$) can have a turning point y_t at which the trajectory is parallel to the x axis ($dy/dx = 0$) if

$$\sigma(y_t) - \sigma(y_0) = -\varphi^2/2. \quad (20)$$

The direction of signal plasmon beam propagation sufficiently depends both on the induced inhomogeneity and on the initial tilt angle φ. For the fixed inhomogeneity the turning point exists only if the initial tilt angle is less than the critical value φ_{cr} determined as:

$$\max(-\sigma) = \varphi_{cr}^2/2. \quad (21)$$

Here we neglected the permittivity change at the initial point y_0 (that is considered to be at a rather large distance from a pump plasmon).

Depending on the ratio between initial tilt signal angle and maximum effective inhomogeneity induces by pump three possible regimes of signal plasmon propagation can be distinguished (see Fig. 1). Notice that although the trajectory itself sufficiently depends on the reference beam profile the regime of propagation does not depend on it.

The first regime is the analogous of ordinary refraction of a beam in the inhomogeneous media and takes place if $\varphi > \varphi_{cr}$. The trajectory of the signal beam is curved though it keeps propagating at the initial angle after passing the inhomogeneity.

The second regime takes place if $\varphi < \varphi_{cr}$ and is analogous to the total internal reflection phenomenon on the interface of two dielectric media. The signal plasmon beam is reflected from the inhomogeneity.

The third regime corresponds to the strict fulfillment of the condition $\varphi = \varphi_{cr}$. It can be shown (see Eq. (18)) that all the derivatives $d^m y/dx^m$ tend to zero near the turning point so that the signal ray asymptotically approaches inhomogeneity maximum.

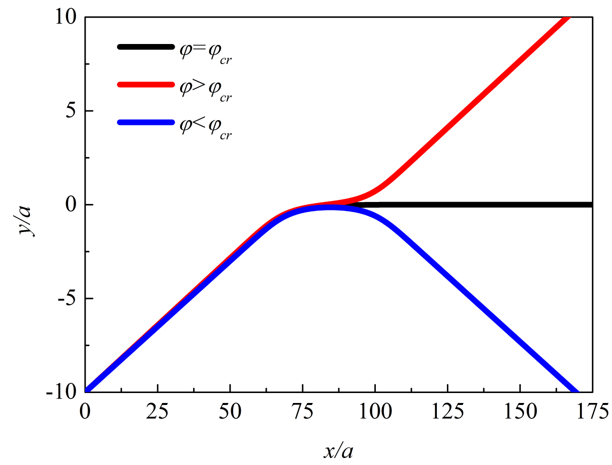

Figure 1: The trajectories of the signal plasmon beam corresponding to the same initial tilt and the different reference plasmon intensities: green curve – below, red curve - equal, blue curve - above the critical value.

The trajectories of the signal plasmon corresponding to these three regimes were calculated by solving the Eq. (18) and are shown in Fig. 1.

A simple mechanical analogy of this effect can be introduced. The ball rolling on a surface can pass over a curved barrier or roll back depending on it initial pulse. Although mathematically one can find the proper pulse for the ball to reach the top of a barrier and stay there this position is unstable and any deviation leads to the ball movement to the one or another side of the barrier. The pump plasmon induced inhomogeneity plays the role of this curved barrier for the signal beam.

5. Spectral approach to the signal SPP propagation description

We confirmed the theoretical results presented here by the numerical simulation of signal SPP beam propagation in the presence of the pump SPP. We neglected the self-action and diffraction of the pump beam and considered its profile unchanged. Such simplification allowed us to solve the Eq. (14) for a signal plasmon beam numerically with a fixed inhomogeneity induced by the pump beam. Hereafter the specification signal of a plasmon beam will be omitted.

The maximum of the initial spatial SPP spectrum $S(x=0,k_y)$:

$$S(x=0,k_y) = \int\limits_{-\infty}^{+\infty} A(x=0,k_y)e^{-ik_y y}dy \quad (22)$$

at certain k_{yi} can be associated with the initial angle φ:

$$\varphi = \frac{k_{yi}}{\beta}. \quad (23)$$

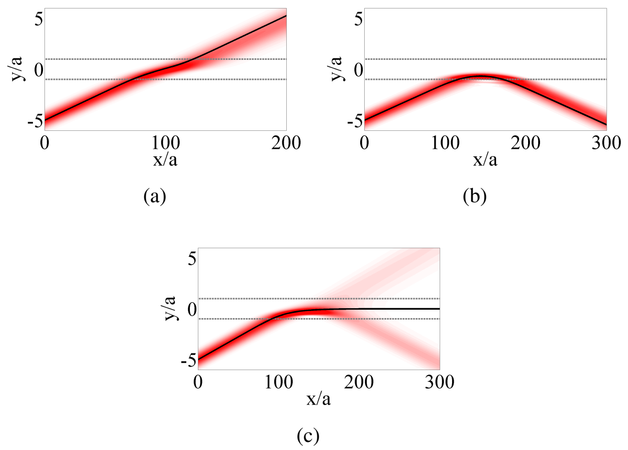

Figure 2: (a) The refraction ($\varphi = 1.25\varphi_{cr}$) and (b) reflection ($\varphi = 0.8\varphi_{cr}$) of SPP on the induced inhomogeneity (shown by gray dash lines). (c) SPP partial reflection and partial refraction near the critical initial angle $\varphi = \varphi_{cr}$. Black line illustrates the results of the ray theory.

For the definiteness we assume that $k_{yi} > 0$ that means that the plasmon beam is initially located in the region $y < 0$. The paraxial approximation used above is valid only if the tilt angles are small $\varphi \ll 1$, or $k_{yi} \ll \beta$.

If all components of the spatial plasmon spectrum correspond to the initial angles of total internal reflection or to the refraction angles the ray theory describes the plasmon propagation with a very good accuracy (see Fig. (2).

However near critical angle due to the finite spatial spectrum width part of the spectral components get in the reflection region while the others occur to be refracted. This two spectral parts are bounded by the critical value k_{y0}. The transmission coefficient can be calculated:

$$T = \frac{\int\limits_{k_{y0}}^{+\infty} S^2(x=0,k_y)dk_y}{\int\limits_{-\infty}^{+\infty} S^2(x=0,k_y)dk_y}. \qquad (24)$$

Note that Fresnel reflection is neglected in our examination since the inhomogeneity is very small $\sigma \ll 1$ and the phenomenon discussed above appear only due to the finite width of the spatial spectrum.

As it was discussed above there is only one spectral component referring to the inhomogeneity-trapped propagation. Therefore the amount of energy corresponding to the propagation parallel to the inhomogeneity exactly equals to zero since it can be defined as: $\int\limits_{k_{y0}}^{k_{y0}} S^2(x = 0, k_y)dk_y = 0$. This approach explains why the beam tilted at near-critical angle is divided by inhomogeneity into two but not three parts (in contrast to the mentioned above three regimes of ray propagation).

Therefore the plasmon beam is partially reflected and partially transmitted through the inhomogeneity if its central spectral component is close to the critical value as it is illustrated by the Fig. 2 (c).

In our numerical calculations the amplitude of the induced inhomogeneity was $\sigma_0 = 10^{-3}$ and its profile had Gaussian form $\sigma(y) = \sigma_0 e^{-y^2/a^2}$ with the width $a = 50\beta/2\pi$ equal to the signal beam width. The value of critical angle was $\varphi_{cr} = 2.6^o$.

6. SPP beam tunneling through the induced inhomogeneity

According to the developed ray theory of signal SPP propagation in the presence of the induced inhomogeneity the regime of the SPP propagation (reflection or refraction) is determined only by the magnitude of the inhomogeneity and not by its shape or width. However if the inhomogeneity width is rather small the effect of tunneling that is similar to the same effect in quantum mechanics can be observed.

In analogy with quantum mechanics (see, for example, [18]) we can determine the turning points $y_{t_{1,2}}$ (see Eq. (20)):

$$\sigma(y_{t_{1,2}}) = -\frac{\varphi^2}{2} \qquad (25)$$

and the tunneling coefficient can be written as:

$$\tilde{T} = \frac{e^{-2\psi}}{\left(1 + \frac{1}{4}e^{-2\psi}\right)^2}, \qquad (26)$$

where

$$\psi = \int\limits_{y_{t_1}}^{y_{t_2}} \beta\theta_1 \sqrt{-(\sigma(y) + \frac{\varphi^2}{2})} dy. \qquad (27)$$

It is important to mention that this expression for tunneling coefficient is different from the transmission coefficient (24). In both cases of tunneling and partial transmission the incident beam is partially reflected and partially refracted nevertheless this phenomenon occurs due to two different mechanisms. Tunneling of the beam through the inhomogeneity is possible due to the leakage of the waves that are evanescent between the turning points and sufficiently depends on the width and profile of the inhomogeneity as is follows from Eq. (26). Partial transmission described by Eq. (24) occurs due to the wide spatial plasmon beam spectrum and depends only on the signal beam shape and the magnitude of the inhomogeneity (that determines k_{y0}).

Fig. 3 illustrates the tunneling of the signal plasmon beam through the inhomogeneities of different width.

In order to determine accurately the transmission and reflection coefficients one should take into account both finite spatial spectrum width and tunneling effect. The resulting transmission coefficient can be found as:

$$T = \int\limits_{-\infty}^{+\infty} \tilde{T}(k_y) S^2(k_y) dk_y. \qquad (28)$$

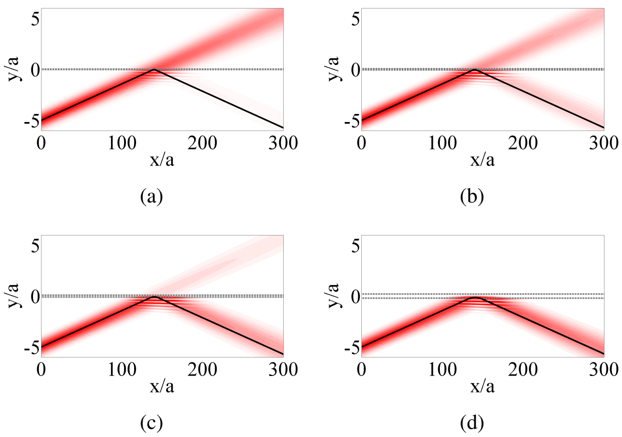

Figure 3: SPP tunneling through the induced inhomogeneity shown by gray lines. Reference beam width equals $a_r = 0.02a_s, 0.05a_s, 0.1a_s, 0.2a_s$ for (a)-(d) figures correspondingly. Black lines illustrate the results of the ray theory.

In order to induce higher inhomogeneities the pump beam can be focused and the signal plasmon can be reflected from its waist. Although we considered the reference beam profile to remain unchanged the results obtained above can be also applicable for this case if the interaction region is rather small. Anyway the tunneling effect certainly will take place for a narrow reference beam waists.

7. Conclusion

The theory of surface plasmon polariton interaction at the interface between metal and dielectric with saturable Kerr nonlinearity was developed. The equation for the self-influence of the reference plasmon beam was derived using the slowly varying amplitude method and averaging over the transversal coordinate. Similar equation was obtained for the weak signal beam propagation in the presence of the inhomogeneity induced by the pump beam. Due to the nonlinearity saturation the profile of the inhomogeneity can differ from the profile of the pump surface plasmon beam.

Using the ray theory we found three regimes of possible signal plasmon propagation: refraction, total internal reflection and degenerate regime of signal trapping. Due to the finite spatial spectrum width signal beam can be almost entirely or partially reflected or refracted. The effect of tunneling of a signal plasmon beam through a narrow inhomogeneity is demonstrated.

Therefore depending on the initial tilt and the inhomogeneity value controlled by the reference plasmon intensity the signal plasmon can be reflected of transmitted through the inhomogeneity. This allows us to manage the signal plasmon direction by modulating the reference plasmon intensity.

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