Exotic Properties and Potential Applications of Quantum Metamaterials

Romain Fleury[1], and Andrea Alù [1*]

[1]Department of Electrical and Computer Engineering, The University of Texas at Austin, Austin, TX, USA
*corresponding author, E-mail: alu@mail.utexas.edu

Abstract

We discuss here potential venues for applications and exotic features of quantum metamaterials. We explore the coupling of conventional electromagnetic metamaterials with quantum emitters and the wave properties of quantum metamaterials obtained by tailoring their effective band structure. We discuss anomalous enhancement effects in the quantum emission properties of individual and collections of small emitters in the presence of metamaterials, as well as matter-wave cloaking and anomalous tunneling phenomena for quantum mechanical waves in artificial materials with exotic band structures.

1. Introduction

The field of metamaterials and plasmonic materials has evolved tremendously in the past few years, expanding into a variety of novel fields and disciplines. Initially based on purely classical concepts, the trend of the last few years has been to consider smaller and more closely spaced nanoparticles, forcing scientists to consider quantum effects for the proper modeling of metamaterials [1]-[2], in particular for shorter wavelengths in fields like optics and plasmonics. In addition, combination of metamaterials with quantum sources and small optical emitters has tremendously expanded, together with the application of optical antennas for which quantum emitters may represent efficient localized power sources [3]. The field of metamaterials is currently mature to include quantum effects, also tailoring the effective band structure of composite materials to produce anomalous propagation properties for matter waves. In this paper, we theoretically discuss potential applications of quantum metamaterials, considering the coupling of small quantum sources with electromagnetic systems, and the tailoring of band structure to produce anomalous quantum effects.

As a first example of our investigations in this area, inspired by electromagnetic metamaterials, we discuss how low-constitutive parameters [4]-[8], compared to those available in nature, may be especially attractive to modify the quantum emission properties of small sources. Zero-permittivity (ENZ) metamaterial channels have been proposed to increase the spontaneous emission of small optical sources within a purely classical analysis [9]. We discuss here how these effects may be even more dramatic than what predicted based on purely classical mechanisms, including the effects of quantum super-radiance in systems that have a large physical area, but a small electrical size, due to the large phase velocity of the modes supported in an ENZ channel.

In addition, we discuss how, by modifying the effective band structure of metamaterials, we may be able to translate and extend established metamaterial effects to matter waves. ENZ tunneling and plasmonic cloaking will be discussed for matter waves, analyzing the potentials of these effects when translated into the quantum arena. To this end, transmission-line modeling of wave propagation in quantum metamaterials will be applied to anomalous constitutive parameters and composite systems, showing that the tools successfully used in conventional metamaterials may be theoretically extended also in the area of quantum metamaterials.

2. Quantum Metamaterials

Metamaterials are artificial materials characterized by a wave interaction not commonly available in nature. They have been mostly applied to electromagnetic and acoustic waves, but recent interest in the extension and application of these concepts to matter waves has been explored in a variety of scenarios. A design mimicking the Veselago's lens in optics has been proposed for matter waves, exploiting an electron focusing effect across a p-n junction in graphene [10]. Total transmission of cold Rubidium atoms through an array of sub-De Broglie wavelength slits has been theoretically demonstrated in [11]. Semiconductor heterostructures have been exploited to predict total transmission for electrons in a layered 1D quantum metamaterial [12]. Cloaking of matter waves using an invariant transformation of the Schrödinger equation has been theoretically demonstrated in [13].
In the case of periodic arrays, which make the vast majority of metamaterial geometries, at frequencies such that the wavelength is long compared to the periodicity of the metamaterial, the equations governing the wave propagation can be homogenized and effective constitutive parameters may be defined. By carefully designing the sub-wavelength periodic structure of the medium, materials with anomalous values of constitutive parameters may be engineered.

Metamaterials are therefore associated with exotic properties, not directly available in nature, like negative refraction, extreme parameters, fast or slow waves, and extraordinary tunneling. These properties make them particularly interesting for a wide range of applications including far-field imaging, sensing and cloaking. In this section we discuss how these concepts may be applied to matter waves and to electromagnetic waves interacting with quantum systems, both aspects belonging to the general area of quantum metamaterials (QMM), whose properties cannot be described within classical concepts.

2.1. Classification and general discussion

2.1.1. Type I quantum metamaterials

As a first type of QMM, we will consider quantum systems embedded in a conventional electromagnetic metamaterial, and strongly coupled to it. Here the quantum nature is brought by the quantum system, which for example may be represented by quantum dots and/or quantum wells. The fundamental idea behind type I QMM is that metamaterials can be exploited to dramatically enhance quantum effects. An example of type 1 QMM would be a system of quantum emitters radiating in an electromagnetic metamaterial. We will treat this type of QMM in section 3 of this paper, and show that very peculiar properties can be achieved when including quantum effects in the classical electrodynamic theory of metamaterials.

2.1.2. Type II quantum metamaterials

The second type of quantum metamaterial (type II QMM) consists of an artificial medium supporting quantum or matter waves. In order to further illustrate this concept, let's review how the motion of particles can be described by a wave equation with effective parameters. The time-independent Schrödinger equation for a particle can be written as

$$\left[-\frac{\hbar^2}{2m_0}\nabla^2+V_c(\vec r)+U(\vec r)\right]\Psi=\left[H_0+U(\vec r)\right]\Psi=E\,\Psi\,, \quad (1)$$

where we have assumed that the potential energy may be split into a periodic part $V_c(\vec r)$ and a non-periodic part $U(\vec r)$, which is assumed to be slowly varying on the scale of the lattice constant. H_0 represents the part of the full Hamiltonian that commutes with all the translation operators constructed from a lattice vector. By virtue of Bloch's theorem, it is possible to find a basis of common eigenvectors of H_0 and all the lattice translation operators. We denote this basis of Bloch's functions as $\Psi_{n\vec k}=e^{i\vec k.\vec r}u_{n\vec k}(\vec r)$, where $u_{n\vec k}(\vec r)$ has the lattice periodicity, and we use it to expand the solution Ψ of the full Schrödinger equation (1) as

$$\Psi=\sum_{n,k}\langle\Psi_{nk}|\Psi\rangle|\Psi_{nk}\rangle\,. \quad (2)$$

It can be shown [14] that around a maximum or minimum in the band diagram, within the single band approximation and assuming that the strength of the potential is small compared with the fundamental bandgap, one term in the expansion (2) is dominant and the solution of equation (1) is in first approximation

$$\Psi=F(\vec r)u_{n0}(\vec r)\,. \quad (3)$$

$F(\vec r)$ is the envelope function and satisfies the single-band effective mass equation:

$$\left[-\frac{\hbar^2}{2}\nabla(\hat m^{*-1}\nabla)+U(\vec r)\right]F(\vec r)=\left(E-E_{c0}\right)F(\vec r) \quad (4)$$

where E_{c0} is the energy at the extremum, and the effective mass tensor is defined as:

$$\hat m^*_{ij}=\hbar^2\frac{\partial^2(E-E_{c0})}{\partial k_i\partial k_j} \quad (5)$$

If the potential energy changes very rapidly (for example, in a heterojunction between two direct band-gap semiconductors), the theory presented above is still valid if one considers the boundary condition [15]:

$$F \text{ and} \left(\hat m^{*-1}\frac{\partial}{\partial\hat n}\right)F \text{ continuous} \quad (6)$$

We see that all the effects from the periodic potential are absorbed into the effective-mass parameter. Considering the external potential U as a medium parameter (i.e., the energy difference at the band edge), we may consider the material to be an effective homogeneous medium for the envelope of the quantum particle. This justifies the term 'quantum metamaterial'.

Examples of type II QMM, for which the single band effective mass approximation is valid, include conduction electrons in direct band gap semiconductors operated close to their Γ point, or cold atoms in an optical lattice. The analogy between type II QMM and electromagnetic or acoustic metamaterials make them particularly suitable for translating and exploiting the most exotic effects discovered in these fields to matter waves. We will discuss specific examples of type II QMM in sections 4 and 5.

3. Boosting quantum super-radiance in an epsilon-near-zero medium

As example of type I QMM we will consider the emission properties of a system of identical 2-level quantum emitters in a metamaterial with extremely low value of effective permittivity. Consider a non-magnetic background medium with effective permittivity ε in which we embed a system of N identical 2-level quantum emitters, radiating at frequency ω_0, for instance quantum dots or atoms. We note N_0 the concentration of emitters, and d the off-diagonal matrix element of the dipole moment operator (chosen to be real). We will assume that at $t=0$ all the 2-levels atoms are in their excited state. All the emitters are therefore expected to spontaneously radiate as a result of the interaction with the quantum fluctuations of the electric field. The background is considered as a classical metamaterial in which one can achieve anomalous values of permittivity at the emission frequency of the 2-level atoms.

As was first pointed out by Dicke in his original paper [16] about coherent super-radiance, the independence of the spontaneous decay of several identical atoms is a wild assumption, and a more accurate description of the problem leads to quite different results. Interestingly, the interaction of the atoms through their radiation electromagnetic fields results in correlation between the atomic dipole moments, leading to the formation of macroscopic polarization, the latter being proportional to the total number of atoms in the system N. As a consequence, the total radiation intensity is considerably enhanced, and the spontaneous decay time is shortened. We shall now describe this effect more quantitatively, and explain how it can be further enhanced by decreasing the permittivity ε of the medium.

3.1. Model

To address the effect of the permittivity of the medium on the radiative properties of our quantum metamaterial we exploit a semi-classical model of super-radiance. This model is suitable for describing systems whose size exceeds the emission wavelength. We briefly review the basic assumptions of this model: we will assume that the system is a rectangular box whose size is smaller than the Arecchi-Courtens length (critical length beyond which the system splits up to several incoherent super-radiating segments) [17]. The system is opened at both ends along the *x* axis. In addition, we assume that all physical quantities depend only on one spatial coordinate x and that the quantum dipole moments and electric fields are polarized along the y direction. This simple model has been proven to describe with excellent accuracy experimentally measured super-radiant pulses [18].

The semi-classical approach that we follow combines the quantum mechanical treatment of the 2-level system with the classical treatment of the radiation field. His main result is the dimensionless Maxwell-Bloch non-linear PDE equations system [19]

$$\frac{\partial R}{\partial \tau'} = \mathrm{Z}\mathrm{E} \qquad (7)$$

$$\frac{\partial \mathrm{Z}}{\partial \tau'} = -R\mathrm{E} \qquad (8)$$

$$\frac{\partial \mathrm{E}}{\partial \xi} = R, \qquad (9)$$

where Z is the population difference or inversion, R is the envelope of the non-diagonal element of the local average of the density matrix, which is linked with the local polarization of the medium, E is the normalized electric field, ξ is the normalized coordinate x and τ' is the normalized retarded time

$$\tau' = \tau - \xi \qquad (10)$$

where τ is the normalized time. The way that these quantities are normalized is of crucial importance to determine the scaling properties of super-radiance and the effect of the background permittivity. We have

$$\tau = t\,\Omega_0, \ \xi = x\,x_0^{-1}, \ \mathrm{E} = e\,e_0^{-1} \qquad (11)$$

where e is the electric field. The frequency Ω_0 gives information on the characteristic time of super-radiance

$$\Omega_0 = \sqrt{\frac{d^2\omega_0 N_0}{2\hbar\varepsilon}} \qquad (12)$$

The characteristic length x_0 is the above-mentioned Arecchi-Courtens length which is given by:

$$x_0 = \frac{c}{\Omega_0} \qquad (13)$$

where c is the phase velocity of light in the considered medium. Note than this length does not depend on the permittivity of the background medium. This implies that it is not possible to extend the spatial range of super-radiance by tailoring ε. This is simply related to the fact that when two emitters are separated by more than x_0, they cannot interact through their radiation field because the electromagnetic interaction cannot go faster than the speed of light in vacuum. As a consequence, they belong to two different uncorrelated super radiating segments. We therefore restrict ourselves to a single super-radiating segment by requiring that the system be smaller than x_0.

The normalization constant for the electric field is:

$$e_0 = \frac{i\Omega_0}{d} \qquad (14)$$

The method for solving this system of equation has first been proposed by Burnham and Chiao when modeling the coherent resonant fluorescence excited by a short light pulse [20]. We consider the auto-modeling solution of the Maxwell-Bloch equation, and numerically solve the resulting differential equation with the appropriate boundary conditions. This yields the magnitude of the electric field as a function of *x* and *t*. From its value, we can compute the radiation intensity (number of photons per unit time radiated through one end of the system).

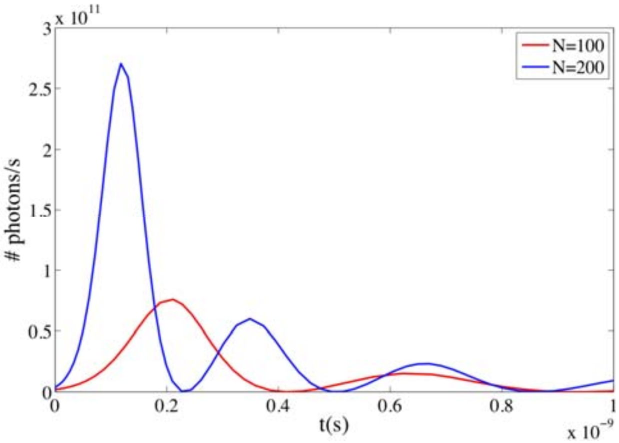

Figure 1: Output intensity versus time for different concentrations of emitters. The relative permittivity of the background medium is 1.

3.2. Results

In order to highlight the exotic features of our quantum metamaterial we plot in Fig. 1 the number of emitted photons versus time for different values of total number of emitters. Fig. 2 compares the output intensity for different values of the background permittivity. The energy levels of the identical quantum emitters are assumed to be separated by 3.1 eV, the dipole moment d is chosen to be 10^{-29} C.m, and the cross-sectional area of the system used in the intensity calculations is 4000 nm^2. We notice that the maximum peak value is proportional to N^2. This is the dramatic effect of interaction between the emitters through their common radiation field. One would indeed expect the output intensity to be proportional to N if the sources were independent. We also see that the super-radiance characteristic time is inversely proportional to the total number of atoms in the system. This suggests that the radiation decay time can be extremely short for sufficiently dense media.

The effect of decreasing the permittivity is also dramatic: both radiation intensity and characteristic time are roughly inversely proportional to $\sqrt{\varepsilon}$. This suggests the use of metamaterials as background media in order to achieve very low values of permittivity and further enhance both the maximum output intensity and the radiation time.

In summary, this very simple model predicts a significant boosting of spontaneous emission in quantum metamaterials made of confined identical quantum emitters in an ENZ background. This result is significantly larger than what predicted in [9] adopting a purely classical analysis.

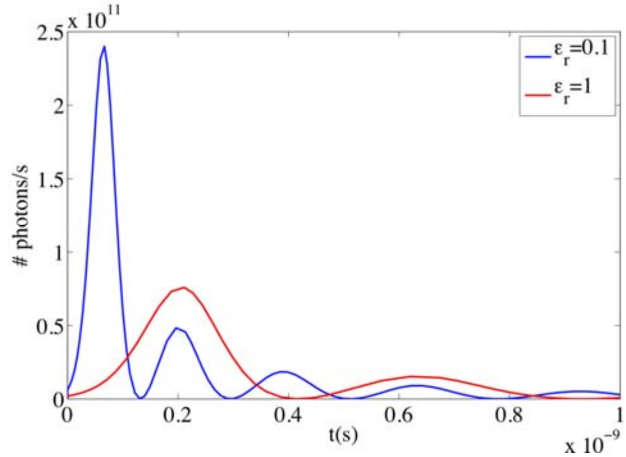

Figure 2: Output intensity versus time for different value of the relative permittivity of the background medium, for 100 emitters.

4. Plasmonic cloaking of matter waves

As an example of a potential application of type-II QMM, we investigate the possibility of cloaking matter waves by cancelling the scattering cross-section of a given potential profile.

4.1. Preliminaries

Cloaking of matter waves has been theoretically predicted for spherical systems with given potential energy and effective mass, by considering an invariant transformation of the Schrödinger equation [13]. Here, we take a different approach, similar to what has been done in the field of plasmonic cloaking. Similar to what proposed for electromagnetic [21] and acoustic waves [22], the idea is to exploit the scattering cancellation based on the negative local polarizability of a cover made of low-permittivity or low-density metamaterial. The purpose of this cover is to cancel the dominant scattering terms in the multipole expansion of the scattered fields. This method is very robust to geometry since it does not rely on a resonance phenomenon. The first experimental realization of a cloaked device for a free-standing 3D object was based on this principle [23]. We discuss now how plasmonic cloaking may be translated to matter waves. For the purpose of clarity we will not consider motion in a periodic potential, and therefore we consider a quantum particle in vacuum impinging on a region with non-zero potential $U(\vec{r})$. This is equivalent to setting V_c to zero in equation (1). This situation corresponds to the usual quantum problem of scattering by a potential. We make the usual assumptions that the impinging particles are spinless and structureless. The potential is not necessarily central, but it is negligible outside a certain action zone Ω.

4.2. Cloaking condition

Let's start from the Lippmann-Schwinger equation in the position representation: the solution of the Schrödinger equation satisfies the following integral equation [24]:

$$\varphi(\vec{r}) = e^{ikz} + \int d^3\vec{r}\,' G_+(\vec{r}-\vec{r}\,')u(\vec{r})\varphi(\vec{r}), \tag{15}$$

where

$$G_+(\vec{r}) = -\frac{1}{4\pi}\frac{e^{ikr}}{r} \tag{16}$$

Is the Green's function associated with the linear operator $\nabla^2 + k^2$ and $u(\vec{r})$ is a normalized potential related to the potential $U(\vec{r})$ by

$$U(\vec{r}) = \frac{\hbar^2}{2m_0}u(\vec{r}). \tag{17}$$

Equation (14) can be solved by iteration, but if the potential $u(\vec{r})$ is small enough one can keep only the first term, following Born approximation

$$\varphi(\vec{r}) = e^{ikz} - \frac{1}{4\pi}\int d^3\vec{r}\,' u(\vec{r}\,')e^{i\vec{K}.\vec{r}\,'}, \tag{18}$$

where $\vec{K} = \vec{k}_s - \vec{k}_i$ is the difference between the scattered wave vector in the direction $\vec{r}\,'$ and the incident wave vector. Imagine now to be able to add a potential cover $u_c(\vec{r})$ around the action zone of the potential. Let's denote by Ω_c the finite domain where the potential cover is non-zero. Equation (15) becomes:

$$\varphi(\vec{r}) = e^{ikz} - \frac{1}{4\pi}\left(\int_\Omega d^3\vec{r}\,' u(\vec{r}\,')e^{i\vec{K}.\vec{r}\,'} + \int_{\Omega_c} d^3\vec{r}\,' u_c(\vec{r}\,')e^{i\vec{K}.\vec{r}\,'}\right) \tag{19}$$

Cloaking is achieved when the integral over the cover cancels the one over the initial potential:

$$\int_{\Omega_c} d^3\vec{r}\,' u_c(\vec{r}\,')e^{i\vec{K}.\vec{r}\,'} = -\int_\Omega d^3\vec{r}\,' u(\vec{r}\,')e^{i\vec{K}.\vec{r}\,'}. \tag{20}$$

For instance, for constant potentials and in the quasistatic limit, one has the simple condition:

$$u_c = -\frac{\Omega}{\Omega_c}u. \tag{21}$$

For spherical constant potentials, if γ denotes the ratio between the shell and the core radii, the cloaking condition becomes

$$\frac{U_c}{U} = -\frac{\gamma^3}{1-\gamma^3}. \tag{22}$$

The analogies with the case of plasmonic cloaking for electromagnetic or acoustic waves are evident. The multipole expansion is replaced by the Born expansion, and the dipole approximation by the Born approximation. Like in the electromagnetic case, the cover needs to have opposite local polarizability to cancel dipolar scattering. Alone, the cover or the core would scatter, but when combined and interfering with each other, they cancel each other, making the obstacle totally transparent to the impinging wave. The cloaking condition is robust to geometry imperfections or fluctuations in the potentials, since it does not rely on a resonance phenomenon. The associated low observability of the cloaked object may find potential application in electronics, sensing, and imaging.

5. Transmission-line theory of guided matter wave

Given the strong analogy between type II QMM and conventional electromagnetic or acoustic metamaterials, we strongly suspect that type-II QMM may be associated with similar anomalous tunneling phenomena as for classical metamaterials. These phenomena are well described by transmission-line (TL) theory. In this section, we first illustrate the need for a TL theory of matter waves. We show that TL theory is the most suitable tool to understand the extraordinary tunneling phenomena associated with electromagnetic metamaterials, through the examples of ENZ supercoupling and tunneling at the plasmonic Brewster angle. Then, we develop a transmission line theory for guided matter waves which may be directly applied to transpose those concepts into the quantum world, paving the way to a variety of exciting applications.

5.1. ENZ supercoupling

ENZ supercoupling is a peculiar transmission phenomenon which occurs between two waveguides of very different cross-sectional areas. Consider an infinite parallel-plate waveguide directly connected to another infinite waveguide with much smaller cross section. Intuitively, one would expect almost total reflection of the TEM waves at the junction between the two waveguides, because of the huge impedance mismatch introduced by the difference in cross-sections. From TL analysis, however, we find the condition for having zero reflection as [6]

$$\frac{h_1}{\sqrt{\varepsilon_1}} = \frac{h_2}{\sqrt{\varepsilon_2}}, \tag{23}$$

where h denotes the waveguide height and ε is the effective permittivity of the medium filling each waveguide. We have assumed that the permeability in the two waveguide is the same. If $h_2 \ll h_1$ and the permittivities are of the same order of magnitude, condition (22) is far from being fulfilled. But interestingly, if ε_2 is near zero, total transmission is paradoxically achieved for a waveguide of infinitely small h_2 [5]. The associated large field enhancement, uniform all along the small waveguide due to the ENZ quasistatic response, is peculiarly independent of its length and shape and has been proposed for novel concepts in light concentration and harvesting [25], sensing [26], boosting molecular emission [9] or optical non-linearity [27].

5.2. Plasmonic Brewster angle

Tunneling of electromagnetic waves at the plasmonic Brewster angle is another example of anomalous tunneling through a very small aperture. Like ENZ tunneling, it relies on impedance matching. Therefore, this phenomenon is best described by transmission line formalism. Consider a metallic screen corrugated by very narrow slits. It has been shown [28] that the impedance mismatch with a normally incident plane wave can be totally compensated if the angle of incidence satisfies the following matching condition:

$$\cos(\theta)=\frac{\beta_s w}{k_0 d}, \tag{24}$$

where β_s is the wave number inside the screen, w is the width of the slits, k_0 is the free space wave number and d is the periodicity of the grating. This phenomenon is of particular interest since the associated ultrabroadband tunneling can span from dc to the visible range for a fixed incidence angle.

5.3. TL theory of guided matter waves

TL theory for matter waves has been proposed to describe propagation of plane waves in one dimensional problems [29]. We propose to extend it to the description of guided matter waves, in order to enable easy transposition of the two above mentioned extraordinary tunneling. Imagine a rectangular matter waveguide made out of infinite potential walls filled up with a type-II QMM with effective parameters m^* and V . The solution of the time-independent Schrödinger equation for the envelope function can be written as

$$\varphi(x,y,z)=\varphi_0\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)e^{ik_z z} \tag{25}$$

where a and b are the rectangular cross-section dimensions in the x and y directions, φ_0 is a constant, and (n,m) is a pair of nonzero integers. We will restrict ourselves to the $(1,1)$ mode, for which the guided wave number k_z satisfies the dispersion relation

$$k_z^2=\frac{2m^*}{\hbar^2}\left(E-V\right)-\left(\frac{\pi}{a}\right)^2-\left(\frac{\pi}{b}\right)^2. \tag{26}$$

We propose the following TL model for the wave propagation along z. The line voltage $u(z)$ is defined as

$$u(z)=\varphi(\frac{a}{2},\frac{a}{2},z)=\varphi_0 e^{ik_z z} \tag{27}$$

and the line current as

$$i(z)=\frac{ab}{m^*}\frac{du}{dz}=iab\frac{\varphi_0}{m^*}k_z e^{ik_z z}. \tag{28}$$

The primary parameters of the line are the shunt line admittance Y_p and the series line impedance Z_s, given by

$$Z_s=-\frac{m^*}{ab} \tag{29}$$

$$Y_p=\frac{ab}{m^*}k_z^2. \tag{30}$$

The line impedance is therefore:

$$\eta=\sqrt{\frac{Z_s}{Y_p}}=i\frac{m^*k_z}{ab} \tag{31}$$

Note that this definition of voltage and current has been chosen to ensure the continuity of the probability and probability current along the line. With these definitions, the junction between two waveguides with different cross-section can be directly modeled by connecting the two TL models together. The condition for total transmission between two semi-infinite waveguides is obtained by equating the line impedances:

$$\frac{m_1^*k_{z1}}{a_1b_1}=\frac{m_2^*k_{z2}}{a_2b_2} \tag{32}$$

It is evident how this theory can be exploited to transpose epsilon-near-zero supercoupling to matter waves, with similar properties as described in Section 5.1. Moreover, this theory is obviously valid for plane waves in unbounded media (or particles with well-defined momentum), written in the form:

$$\varphi(x,y,z)=\varphi_0 e^{ik_z z} \tag{33}$$

An angle of incidence can be taken into account by appropriately inserting factors $\cos\theta_i$ in the definitions of the line voltage and current in order to ensure the continuity of the probability and probability current at the junctions between unbounded media and waveguides. This modification allows a translation of the plasmonic Brewster tunneling to matter waves, which we will explore in detail in an upcoming publication.

6. Conclusions

We have presented and theoretically explored exotic properties and applications of quantum metamaterials. Type I quantum metamaterials, which are constituted of a quantum system strongly coupled to a classical metamaterial, were illustrated by considering the spontaneous radiation properties of a system of identical quantum emitters in a near-zero permittivity medium. We have presented quantitative arguments to support the fact that super-radiance may be strongly enhanced in an epsilon-near-zero background. Type II quantum metamaterials are artificial media with exotic effective properties for matter waves. We have shown how such media can be constructed by tailoring the band structure of quantum particles in periodic potentials. Exotic properties of matter wave metamaterials have been illustrated through several promising examples. We have shown that in principle it is possible to achieve plasmonic cloaking for matter waves, by designing a cover to cancel the scattering from a region of non-zero potential. In addition, we have shown that the applications of classical metamaterials may be successfully translated into the quantum arena. A transmission-line theory for matter waves has been presented and we have showed how this tool may be successfully applied to transpose ENZ supercoupling and plasmonic Brewster angle tunneling to the quantum wave functions. These phenomena may be of interest in a variety of applications including sensing, electronics, or imaging.

Acknowledgements

This work has been supported by the ONR MURI grant No. N00014-10-1-0942.

References

[1] N. I. Zheludev, The road ahead for metamaterials, *Science* 328: 582–583, 2010.

[2] J. Zuloaga, E. Prodan, and P. Nordlander, Quantum plasmonics: optical properties and tunability of metallic nanorods, *ACS Nano* 4: 5269–5276, 2010.

[3] J.J. Greffet, M. Laroche and F. Marquier, Impedance of a nanoantenna and a single quantum emitter, *Phys. Rev. Lett.* 105: 117701, 2010.

[4] A. Alù, M.G. Silveirinha, A. Salandrino and N. Engheta, Epsilon-near-zero metamaterials and electromagnetic sources: tailoring the radiation phase pattern, *Phys. Rev. B* 75: 155410, 2007.

[5] M. G. Silveirinha, and N. Engheta, Tunneling of electromagnetic energy through subwavelength channels and bends using ε-near-zero materials, *Phys. Rev. Lett.* 97: 157403, 2006.

[6] A. Alù, M.G. Silveirinha, and N. Engheta, Transmission-line analysis of ε-near-zero–filled narrow channels, *Phys. Rev. E* 78: 016604, 2008.

[7] R. W. Ziolkowski, Propagation in and scattering from a matched metamaterial having a zero index of refraction, *Phys. Rev. E* 70: 046608, 2004.

[8] B. Edwards, A. Alù, M.G. Silveirinha, and N. Engheta, Experimental verification of plasmonic cloaking at microwave frequencies with metamaterials, *Phys. Rev. Lett.* 103: 153901, 2009.

[9] A. Alù and N. Engheta, Boosting molecular fluorescence with a plasmonic nanolauncher, *Phys. Rev. Lett.*103: 043902, 2009.

[10] V.V. Cheianov, V. Fal'ko and B.L. Altshuler, The focusing of electron flow and a veselago lens in grapheme p-n junctions, *Science* 315:1252-1255, 2007.

[11] E. Moreno, A.I. Fernandez-Dominguez, J. Ignacio Cirac and L. Martin-Moreno, Resonant transmission of cold atoms through subwavelength apertures, *Phys. Rev. Lett.* 95: 170406, 2005.

[12] L. Jenilek, J.D. Baena, J. Voves and R. Marques, Metamaterial inspired perfect-tunneling in semiconductors heterostructures, *New Journal of Physics* 13: 083011, 2011.

[13] S. Zhang, D.A. Genov, C. Sun and X. Zhang, Cloaking of matter waves, *Phys. Rev. Lett.* 100:123002, 2008.

[14] J.M. Luttinger and W. Kohn, Motion of electrons and holes in perturbed periodic fields, *Phys. Rev.* 97: 869–883, 1955.

[15] L.C. Lew Yan Voon, M. Willatzen, *The k.p method, Electronic properties of semiconductors*, Springer, New York, 2009.

[16] R.H. Dicke, Coherence in spontaneous radiation processes, *Phys. Rev.* 93: 99–110, 1954.

[17] F.T. Arecchi and E. Courtens, Cooperative phenomena in resonant electromagnetic propagation, *Phys. Rev. A* 2: 1730-1737, 1970.

[18] N.Skribanowitz, I.P. Herman, J.C. MacGillivray and M.S. Feld, Observation of Dicke super-radiance in optically pumped HF gas, *Phys. Rev. Lett.* 30:309-312, 1973.

[19] M.G. Benedict, A.M. Ermolaev, V.A. Malyshev, I.V. Solokov and E.D Trifonov, *Super-radiance, multiatomic coherent emission*, Institute of Physics Publishing, Bristol, 1996.

[20] D.C. Burnham and R.Y. Chiao, Coherent resonance fluorescence excited by short light pulses, *Phys. Rev.* 178: 2025–2035, 1969.

[21] A. Alù and N. Engheta, Achieving transparency with plasmonic and metamaterial coatings, *Phys. Rev. E* 72: 016623, 2005.

[22] M. Guild, A .Alù and M.R. Haberman, Cancellation of acoustic scattering from an elastic sphere, *J. Acoust. Soc. Am*, 129:1355-1365, 2011.

[23] D. Rainwater, A. Kerkhoff, K. Melin, J. C. Soric, G. Moreno and A. Alù, Experimental verification of three-dimensional plasmonic cloaking in free space, *New Journal of Physics*, 14:013054, 2012.

[24] A. Bohm, *Quantum Mechanics, Foundations and Applications*, 3rd edition, Springer-Verlag, New York, 1994.

[25] A. Alù and N. Engheta, Light squeezing through arbitrarily shaped plasmonic channels and sharp bends, *Phys. Rev. B* 78: 035440, 2008.

[26] A. Alù and N. Engheta, Dielectric sensing in ε-near zero narrow waveguide channels, *Phys. Rev. B* 78: 045102, 2008.

[27] C. Argyropoulos, P.Y. Chen, G. D'Aguanno, N. Engheta and A. Alù, Boosting optical non-linearities in epsilon-near-zero plasmonic channels, *Phys. Rev. B* 85: 045129, 2012.

[28] A. Alù, G. D'aguanno, N. Mattiucci and M.J. Bloemer, Plasmonic Brewster angle: broadband extroadinary transmission through optical gratings, *Phys. Rev. Lett.* 106:123902, 2011.

[29] A.N. Khonder, M. Rezwan, A.F.M. Anwar, Transmission line analogy of resonance tunneling phenomena: The generalized impedance concept, *Journal of Applied physics*, 63:5191-5193, 1988.