Phase constant peculiarities of cylindrical zero-index anisotropic metamaterial waveguide.

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Abstract

Here we present the phase constant dependencies of propagating eigenmodes of open cylindrical anisotropic metamaterial waveguide when the metamaterial permittivity and permeability tensor components may accept values close or equal to zero. Dispersion characteristics of rod and hollow-core waveguides with the radii 0.5, 2.5 and 5 mm at the left handed polarization of microwave are shown here. There are unusual shape of eigenmode dispersion characteristics and anomalous sectors of the characteristics at certain frequencies. The first eigenmode of rod waveguide with the lowest cutoff frequency is a particularly important mode because it is a single one in the frequency range 1.0–1.9 GHz and some small variations on the frequency produce large changes in the phase constant. We can observe packages of dispersion characteristic branches when their cutoff frequencies closed to the metamaterial electric and magnetic plasma frequencies between 1.9 and 3.5 GHz. There are only three modes in the hollow core anisotropic metamaterial waveguide at the frequency range 1.4–2.8 GHz.

1. Introduction

In the last decade many specialist focused on the experimental and theoretical investigations of the zero-refractive index (or zero-index) metamaterials. The metamaterials attractive to researches due to their unconventional constitutive parameters and different anomalous effects too. Zero-index metamaterials may have the epsilon-near-zero (ENZ) and mu-near-zero (MNZ) properties simultaneously or one after another at different frequencies. These metamaterials are used in different devices as a transformer to achieve the perfect impedance match between two waveguides with a negligible reflection or to improve the electromagnetic (EM) wave transmission through a waveguide bend, for the matching of waveguide structure impedance with the free space impedance and etc. The metamaterials provides manipulating of the antenna phase fronts and enhancing the antenna radiation directivity. In a Zero-index metamaterial waveguide can be observed a super-tunneling effect. ENZ metamaterials may allow reducing of waveguide sizes and can be used as a frequency selective surface [1–5].

The controllable devices as modulators, phaseshifters, shields and etc. can be created on the base of anisotropic materials [6].

Zero-index metamaterials are dispersive media. The constitutive parameters of anisotropic metamaterials can be described by expressions that involve the plasma frequencies. A waveguide that has a boundary of anisotropic metamaterial-dielectric (air) can be assigned to plasmonic waveguides. Here we present dispersion characteristics of two open plasmonic waveguides Fig. 1.

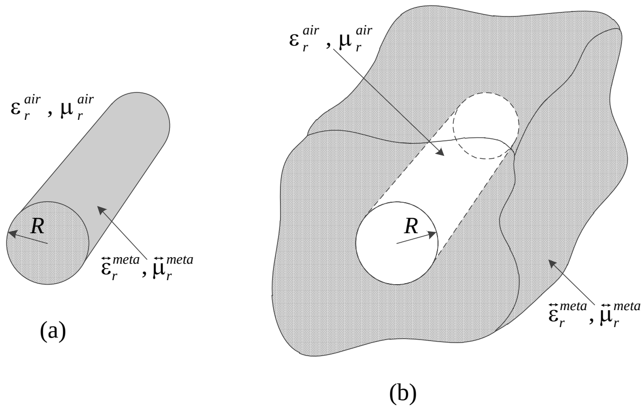

Figure 1: Cylindrical zero-index anisotropic metamaterial waveguide model (a) – open waveguide; (b) – hollow-core waveguide.

2. Permittivity and permeability tensors of zero-index anisotropic metamaterial

Electrodynamical parameters of the uniaxial electrically and magnetically anisotropic metamaterial, characterized by relative permittivity $\overleftrightarrow{\varepsilon}_r^{meta}$ and permeability $\overleftrightarrow{\mu}_r^{meta}$ tensors (1), were taken from the article [7]. In the mentioned article was considered an anisotropic dispersive lossless metamaterial slab. For this reason there were given only the real parts of the relative permittivity (ε_{xx}, ε_{zz}) and relative permeability (μ_{xx}, μ_{zz}) tensor components:

$$\overleftrightarrow{\varepsilon}_r^{meta} = \begin{vmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{vmatrix}, \; \overleftrightarrow{\mu}_r^{meta} = \begin{vmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{xx} & 0 \\ 0 & 0 & \mu_{zz} \end{vmatrix}. \quad (1)$$

The tensor components of the relative permittivity and the relative permeability are described by following formulas:

$$\varepsilon_{xx} = 1 - \omega_{epxx}^2 / \omega^2; \qquad \varepsilon_{zz} = 1 - \omega_{epzz}^2 / \omega^2;$$
$$\mu_{xx} = 1 - \omega_{epxx}^2 / \omega^2; \qquad \mu_{zz} = 1 - \omega_{epzz}^2 / \omega^2, \tag{2}$$

where $\omega = 2\pi f$ – angular frequency of EM oscillation; metamaterial electric $f_{epxx} = 3.46$ GHz, $f_{epzz} = 2.5$ GHz and magnetic $f_{mpxx} = 2.45$ GHz, $f_{mpzz} = 2$ GHz plasma frequencies, taken from [7].

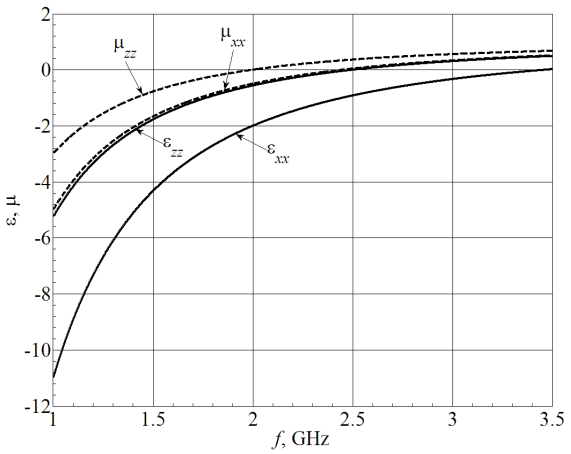

Figure 2: Dependences of the relative permittivity and permeability tensor components of the metamaterial on the frequency.

In Fig. 2 are presented dependencies of tensor components of the relative permittivity ε_{xx}, ε_{zz} and the relative permeability μ_{xx}, μ_{zz}.

The permittivity components ε_{xx} and ε_{zz} have negative values from 1.0 to ~3.5 GHz and from 1.0 to ~2.5 GHz, respectively. The permeability components μ_{xx} and μ_{zz} have negative values from 1.0 to ~2.5 GHz and from 1.0 to ~2 GHz, respectively. All tensor components are negative at the frequency range from 1.0 GHz to ~2 GHz. Absolute values of tensor components are less than 1 at the frequency range from ~2.5 GHz to 4 GHz. The values of tensor components become equal to zero at the operating frequency f equal to the metamaterial electric or magnetic plasma frequencies. This metamaterial is a plasmonic one.

3. Dispersion characteristics of open cylindrical zero-index anisotropic metamaterial waveguide.

The solution of Maxwell's equations for the circular anisotropic metamaterial waveguide was carried out by the partial area method [8–10]. The computer program for the dispersion characteristic calculations has created in MATLAB language. Our computer program allows take into account a very large material attenuation as well as the values of non-diagonal tensor components [9, 10].

We present here how the radius value of plasmonic waveguides affects on the propagating eigenmodes' dispersion characteristics, including dependencies of the eigenmode quantity and mode cutoff frequencies.

In Figs. 3, 4 and 5 are shown dispersion characteristics (phase constants) of open cylindrical waveguide (Fig. 1a) made of the uniaxial electrically and magnetically anisotropic metamaterial in the frequency range 1–3.5 GHz.

The calculations are performed for EM waves with left-handed circular polarization ($e^{+im\varphi}$), where m = 1 is azimuthal symmetry index, φ is the azimuthal coordinate.

Here are shown the phase constant h′ (the real part of longitudinal propagation constant) dependencies of plasmonic metamaterial waveguides with radii R equal to 0.5 mm, 2.5 mm, and 5 mm. The phase constant h′ is equal to $2\pi/\lambda_w$, where λ_w is the wavelength of certain mode. The analysis of Figs 3–5 shows that there are three main frequency areas where localize dispersion curves. A shape of all dispersion characteristics are unusual in the comparison with traditional dispersion characteristics of open cylindrical waveguides made of dielectrics, semiconductors or gyroelectric plasma [8–10]. Because the dispersion characteristic branches of analyzed here waveguides are quite vertical.

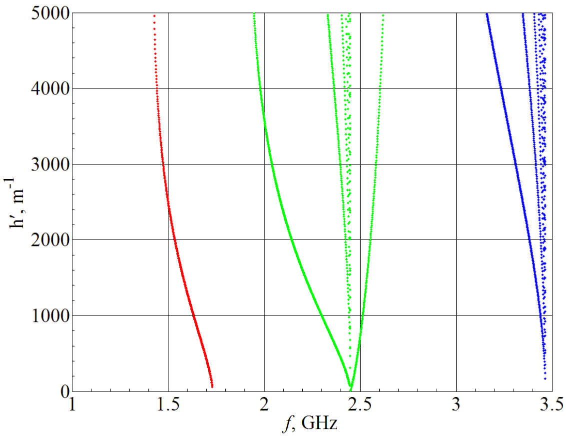

Figure 3: Phase constant dependencies of propagating eigenmodes of the anisotropic metamaterial waveguide with R = 0.5 mm.

We see that there is a single mode with the cutoff frequency 1.73 GHz, 1.71 GHz and 1.63 GHz, when waveguide radius is accordingly 0.5 mm, 2.5 mm and 5 mm (Figs. 3–5, red curves). The cutoff frequency of this mode shifted in the direction of lower frequencies with increasing of the waveguide radius. This first single mode is special one because the mode does not match any of plasma f_{epxx}, f_{epzz}, f_{mpxx}, f_{mpzz} frequencies. We can observe how a shape of the dispersion characteristic changes in the vicinity of the cutoff frequency.

We would like to draw your attention to the fact that the anisotropic metamaterial is described by the negative tensor components ε_{xx}, ε_{zz}, μ_{xx}, μ_{zz} in the frequencies less than 2 GHz (see Fig. 2). It is mean that the first mode propagates in the waveguide when the metamaterial is double negative (DN). This single mode (red curve in Figs 3–5) is particularly important because small changes in frequency produce large changes in the phase constant. The mode can be used for worked out a sensitive narrowband phaseshifter at frequencies between 1.4 and 1.65 GHz (Fig. 3) or other potential microwave devices.

We can watch a package with dispersion branches closed to cutoff frequency 2.5 GHz (Figs 3–5, green curves). We see that the left lateral dispersion branch of the package is a special eigenmode, i.e. this one is separated by a larger distance from other eigenmodes. The vertical branch of the left lateral mode is located on the magnetic plasma f_{mpzz} frequency equal to 2 GHz. We can distinguish also the right lateral dispersion branch of the package.

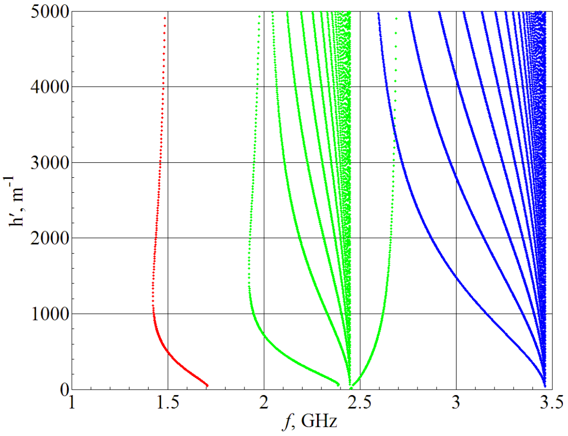

Figure 4: Phase constant dependencies of propagating eigenmodes of the anisotropic metamaterial waveguide with $R = 2.5$ mm.

The mode with this dispersion characteristic is also more specific one. i.e. this mode is separated by a larger distance from other modes. The vertical branch of this mode is located about 2.7 GHz and shifted at the higher frequencies with increasing of a radius.

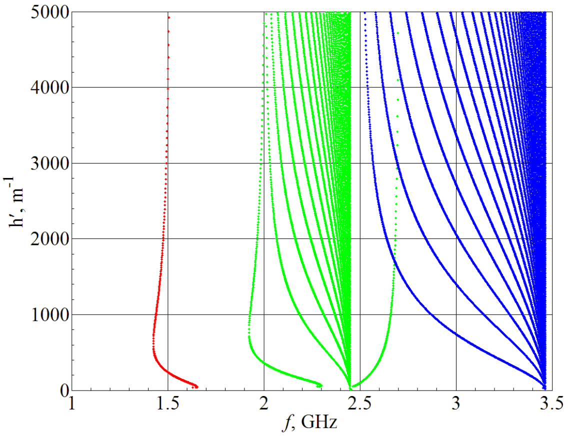

Figure 5: Phase constant dependencies of propagating eigenmodes of the anisotropic metamaterial waveguide with $R = 5$ mm.

A dense bunch of dispersion curves located between the extreme left and right curves that were described before. The number of curves increases rapidly at increasing of waveguide radius. It is interesting to note that all dispersion branches of the dense bunch are within the frequency band of 2–2.5 GHz. Apparently the dense bunch of dispersion characteristics related to plasma f_{epzz}, f_{mpxx} frequencies. The cutoff frequencies of dispersion characteristics of the dense bunch are the same and equal to $f \sim 2.46$ GHz. The dispersion curves fan out from a single point f_{mpxx}.

Second dense bunch of dispersion curves is at the electric plasma frequency $f_{epxx} \sim 3.46$ GHz (Figs. 3–5, blue curves). The number of curves increases rapidly at increasing of waveguide radius. All dispersion characteristics are within the frequency band of 2.5 GHz and 3.46 GHz.

The greatest number of modes can be excited at the electric plasma frequency $f_{epxx} \sim 3.46$ GHz in the comparison with other plasma frequencies. The cutoff frequencies of dispersion characteristics of this dense bunch are the same and equal to $f \sim 3.46$ GHz.

We did not find the plasmonic metamaterial waveguide eigenmodes in the frequency range from 3.5 GHz till 2000 GHz. The mode absence at higher frequencies is possible to explain by a fact that the metamaterial relative permittivity and permeability values at higher frequencies are close to the ones of air.

4. Dispersion characteristics of hollow-core cylindrical zero-index anisotropic metamaterial waveguide.

In Figs. 6–8 are shown dispersion characteristics (phase constants) of hollow-core cylindrical waveguide (Fig. 1b) made of the uniaxial electrically and magnetically anisotropic metamaterial in the frequency range 1.4–2.8 GHz.

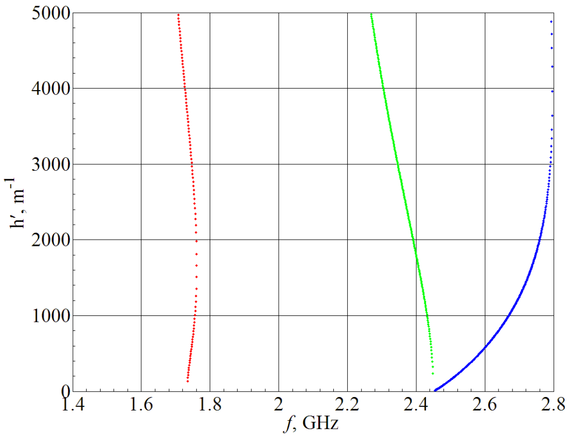

Figure 6: Phase constant dependencies of propagating eigenmodes of the hollow-core anisotropic metamaterial waveguide with $R = 0.5$ mm.

As in the case of an open waveguide there is a single mode that does not match any of plasma frequencies and propagates in the waveguide when the metamaterial is double negative (Figs 6–8, red curves).

The cutoff frequencies of this mode are 1.73 GHz, 1.75 GHz and 1.79 GHz, when waveguide radius accordingly equal to 0.5 mm, 2.5 mm and 5 mm.

We see, that in case of hollow-core waveguide, there are propagating only three modes in frequency range 1.4–2.8 GHz. The branch of second and third modes (Figs. 6–8, green and blue curves) has the same cutoff frequency equal to magnetic plasma frequency $f_{mpxx} = 2.45$ GHz. This cutoff frequency is independent on the waveguide radius. We can observe changes in a shape of dispersion curves with changing of waveguide radius. The dispersion characteristics are more vertical with low values of waveguide radius.

We did not find the plasmonic hollow-core metamaterial waveguide eigenmodes in the any other frequency range. The number of modes propagating in the hollow-core metamaterial waveguide is independent on waveguide radii.

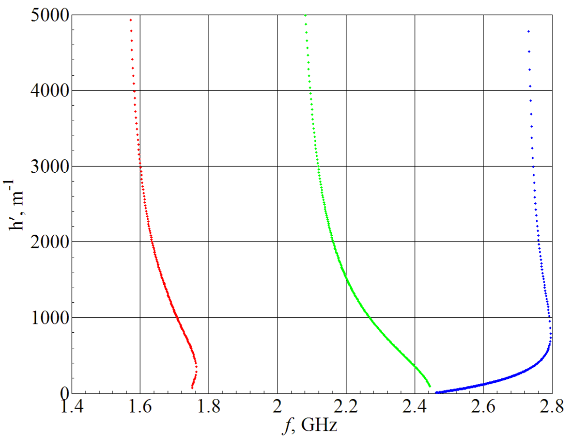

Figure 7: Phase constant dependencies of propagating eigenmodes of the hollow-core anisotropic metamaterial waveguide with $R = 2.5$ mm.

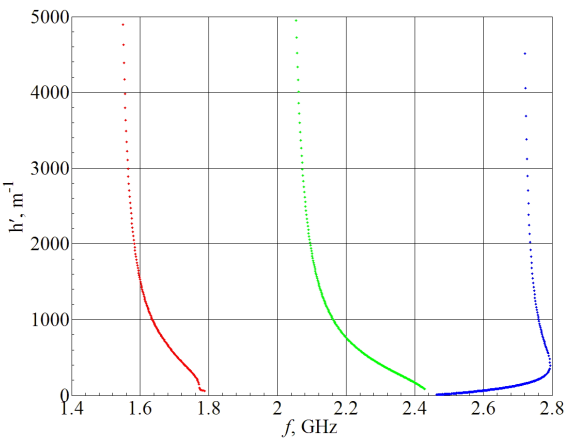

Figure. 8: phase constant dependencies of propagating eigenmodes of the hollow-core anisotropic metamaterial waveguide with $R = 5$ mm.

The specific dispersion characteristic features of hollow anisotropic waveguides can be used for a transmission of laser radiations or to working out plasma wakefield accelerators [11].

Conclusions

1. The open rod and hollow-core cylindrical anisotropic metamaterial waveguides were investigated by using of our MATLAB computer programs based on the partial area method.

2. The anomalous dispersion dependencies are observed for eigenmodes of the waveguides at the metamaterial plasma frequency range between 1 and 3.5 GHz. There are dispersion curve segments for considered waveguides when an increase of frequency accompanied by a decrease of phase constant.

3. We find a single eigenmode of rod waveguide with the cutoff frequency f_{cut} close to 1.7 GHz. The mode has a quite vertical dispersion characteristic when some small variations of f produces the very large changes in the phase constant.

4. There are two dispersion characteristic packages of rod waveguide at the frequency range 1.9–3.5 GHz.

5. There are only three modes in hollow-core waveguides: single mode with $f_{cut} \sim 1.75$ GHz and two modes with f_{cut} equal to the magnetic metamaterial plasma frequency $f_{mpxx} = 2.45$ GHz.

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